Heating up the Baryonic Branch with U-duality: a unified picture of conifold black holes

Elena Cáceres[*1], Carlos Núñez[†2] and Leopoldo A. Pando Zayas [**3]

Facultad de Ciencias
Universidad de Colima
Bernal Diaz del Castillo 340, Colima, México.
and
Theory Group, Department of Physics,
University of Texas at Austin, Austin, TX 78727, USA.

[†] *Department of Physics*
University of Swansea, Singleton Park
Swansea SA2 8PP
United Kingdom.

[**] *Michigan Center for Theoretical Physics*
Randall Laboratory of Physics, the University of Michigan
Ann Arbor, MI 48109-1040. USA

Abstract

We study different aspects of a U-duality recently presented by Maldacena and Martelli and apply it to non-extremal backgrounds. In particular, starting from new non-extremal wrapped D5 branes we generate new non-extremal generalizations of the Baryonic Branch of the Klebanov-Strassler solution. We also elaborate on different conceptual aspects of these U-dualities, like its action on (extremal and non-extremal) Dp branes, dual models for Yang-Mills-like theories, generic asymptotics and decoupling limit of the generated solutions.

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[1]elenac@zippy.ph.utexas.edu

[2]c.nunez@swansea.ac.uk

[3]lpandoz@umich.edu

Contents

1 Introduction

The Maldacena conjecture [1] and some of its refinements [2] are the guiding principle behind much of the progress in the interface String Theory-Quantum Field theory in the last twelve years. The influence of this approach extends to toy models with different number of supersymmetries (SUSY's), systems at finite temperature and/or finite density and lower dimensional systems. The applications to physically relevant systems do not seem to be exhausted and on the contrary, increase with time. In this sense, finding new (trustable) solutions to the equations of motion of String Theory (even in the point particle and classical approximation) has become an industry with various applications in Physics and Mathematics. Solution generating techniques have certainly played a role. Some examples worth mentioning are the combination of T-dualities and shifts of coordinates that generated solutions dual to minimally SUSY superconformal field theories (beta deformations) or duals to non-relativistic field theories [3][1].

In this paper we will focus our attention on a particular solution generating technique that was presented in [6]. The procedure suggested by these authors starts by taking a solution in type IIB string theory proposed to be dual to (a suitably UV-completed version of) $\mathcal{N} = 1$ Super Yang-Mills in four dimensions. The solution has the topology $R^{1,3} \times M_6$ where M_6 preserves four supercharges. The algorithm to generate the new solutions goes as follows: first apply a set of three T-dualities in the R^3 directions, which leaves us with a IIA configuration; lift this configuration to M-theory and boost–with rapidity β–in the eleventh direction; then reduce to IIA and T-dualize back in R^3. This generates a solution that roughly speaking is the dual description to the baryonic branch [7] of the Klebanov-Strassler field theory [8]. For all the technical details of this procedure the reader can refer to [6] or to our Appendix E.

The 'seed solution' (as we will refer to the initial solution on which the generating algorithm is applied) was discussed in the papers [9] (Section 8) and [10] (Section 4.3). In a particular limit, this becomes the exact solution discussed in [11].

We will also refer to the solution generating algorithm, that is a U-duality, as 'rotation' in a sense explained below and in previous papers. In the following we will emphasize some aspects of the generating technique presented in [6] that we find particularly interesting and have not been explicitly discussed in the existing bibliography:

- In the case of [6], the rotation generates D3 brane charge. This is the reason why the $SO(1,3)$ isometry of the background is untouched, in spite of doing *different* operations in time (boost) and in the R^3 directions (T-dualities). As a by-product of the generation of D3 branes the dilaton is invariant under this set of operations.

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[1]In other areas of Physics, solution generating techniques play a major role. For example, many of the interesting solutions in higher dimensional gravity were constructed using solution generating algorithms. See for example [4] for original papers and [5] for a nice summary with an important application.

- It can be seen that this U-duality (sequence of T-dualities, lifts and boosts) described above is equivalent to a *particular case* of 'rotation' in the $SU(3)$ structure of the manifold M_6 which is characterized by a complex 3-form Ω_3 and a 2-form J_2 - see the papers [12] and [13]. This is another way of understanding the presence of the $SO(1,3)$ isometry: from this perspective, all the rotation 'occurs' in the internal M_6.

- The rotation generates a solution that contains two free parameters, the value of the dilaton at infinity (already present in the seed solution) and the boost parameter β. In order for the final background to be dual to the Klebanov-Strassler QFT *decoupled* from gravity, we must take the limit $\beta \to \infty$ so that the generated warp factor vanishes asymptotically. In this sense, the seed solution describes a field theory coupled to gravity. Only in a particular limit-see the discussion in [6], [13], we approach the near brane solution of [11].

- This rotation or U-duality, is a very curious operation from the viewpoint of the dual field theory. Indeed, it generates (aside from the bifundamental fields) new global symmetries, like the baryonic $U(1)$.

- In the same vein: the connection between the two field theories from a geometric viewpoint (Field theory at strong coupling!) is just the mentioned U-duality starting from the $D5$-brane side. From a (weakly interacting) field theoretic viewpoint the operation is very subtle [6]. Indeed, it involves a second order fluctuation expansion (mass spectrum) in the classical and weakly coupled F and D term equations on the quiver side [15], that is matched with a classical and weakly coupled expansion of the twisted compactification of the theory in the wrapped D5 brane mass spectrum [16].

- After the first set of T-dualities described above, we have a IIA background with D2 brane charges. When lifted to M-theory we have M2 branes, that after boosted generate M2 charge and an M-wave. When reduced, this generates a D0, D2 and NS H_3 field. The generation of the H_3 in the presence of RR three-form implies the need to generate F_5. See Appendix E for technical details.

- We emphasized that the U-duality (chain of T-dualities, lifts and boost) described above is a *particular case* of the rotation of J_2, Ω_3 discussed in [12], [13], [14]. On the other hand, the rotation of J_2, Ω_3 relies on the background being supersymmetric, while the U-duality can be also applied to Non-SUSY backgrounds. This will play an important role in the rest of this paper.

1.1   General Idea of this Paper

In this paper we will try to gain a different perspective on this solution generating technique. We will not focus on the rotation of J, Ω but rely on the U-duality description (in spite of the

latter being a particular case of the former). Our interests will be two-fold. On one hand, we will try to get a better handle on the generating algorithm by changing it (suggesting other related algorithms) and applying it to various cases. On the other hand, we will apply it to backgrounds that do not preserve SUSY. We will study the effect of the rotation on black hole solutions, generating *new* non-extremal solutions with horizons and other curious features that will be discussed. We will also start the study of the properties of those newly generated solutions.

This paper is organized as follows: In sections 2 and 2.1 as a warm-up exercise we discuss a chain of dualities and boost acting on non-extremal Dp branes. In section 2.1.1 we will apply this to rotate a background dual to a version of Yang-Mills. Then, in section 2.2 we will study a variation of this U-duality that will clarify various aspects of the coming material. In Section 3 we will U-dualize a new solution describing the non-extremal deformation of a stack of N_c D5 branes wrapping a two-cycle inside the resolved conifold. We will elaborate upon various aspects of this particular rotation in section 3.5. Finally in Section 4 we comment on decoupling aspects of the rotation. Various appendices complement our presentation. They have been written with plenty of detail hoping that colleagues working on these topics will find them useful. We close with a summary, conclusions and a list of possible future projects.

2   A warm-up example: Rotation of Dp-branes

In this section, we will start with a simple example of the 'rotation' procedure. That is, a sequence of T-dualities, bringing the background to a type IIA solution of the supergravity equations of motion, followed by a lift to eleven dimensions, where we will boost the configuration. We will then reduce to IIA and T-dualize back, to what we will call the 'rotated background'. As a toy example, in this section we will rotate flat Dp branes; first in p directions and then in $p - q$ directions. We will do this in detail to appreciate the differences this introduces in the generated solution. In Appendix D, we will propose and analyze another possible U-duality to generate new solutions.

2.1   "Rotation" in p directions

Consider backgrounds of IIA/IIB of the form,

$$ds^2 = H(\rho)^{-1/2}\Big[- h(\rho)dt^2 + d\vec{x}_p^2\Big] + H(\rho)^{1/2}\Big[\frac{d\rho^2}{h(\rho)} + \rho^2 d\Omega_{8-p}\Big],$$

$$e^{2\phi[initial]} = e^{2\phi(\infty)} H^{\frac{3-p}{2}},$$

$$*F_{p+2} = \mathcal{Q}\, Vol\, _{\Omega_{8-p}},$$

$$F_{p+2} = -\partial_\rho A(\rho) dt \wedge dx_1 \wedge .... \wedge dx_p \wedge d\rho \tag{2.1}$$

with,

$$H(\rho) = 1 + \left(\frac{L_p}{\rho}\right)^{7-p}, \quad h(\rho) = 1 - \left(\frac{R_T}{\rho}\right)^{7-p}, \quad A(\rho) = \frac{\alpha}{g_s H(\rho)}, \tag{2.2}$$

where $\alpha = \frac{\tilde{Q}}{L_p^{7-p}}$ and $\mathcal{Q} = (7-p)\tilde{Q}$ is related to the charge of the solution (see Appendix B for details). If we choose $R_T = 0$, the configuration above is typically singular (except for $p = 3$) and preserves 16 SUSYs. Recall that for even (odd) values of p, we are dealing with a solution of Type IIA(B) supergravity.

Now, we will 'rotate' this background, by first T-dualizing in the p directions, this will bring us to a IIA solution, we will lift the solution to eleven-dimensional supergravity and perform a boost of rapidity β in the eleventh-direction. We will then reduce to IIA and T-dualize back in the $\vec{x}_p$ coordinates to obtain what we call the 'rotated background'. Our final rotated background is given by[2],

$$ds^2 = \frac{H(\rho)^{-1/2}}{R(\rho)}\left[-h(\rho)dt^2 + d\vec{x}_p^2\right] + R(\rho)H(\rho)^{1/2}\left[\frac{d\rho^2}{h(\rho)} + \rho^2 d\Omega_{8-p}\right] \tag{2.3}$$

$$e^{2\phi[final]} = e^{2\phi[initial]}R(\rho)^{3-p} = e^{2\phi(\infty)}(H(\rho)R(\rho)^2)^{\frac{3-p}{2}} \tag{2.4}$$

$$F_{p+2} = -\partial_\rho a(\rho)dt \wedge dx_1 \wedge .... \wedge dx_p \wedge d\rho \tag{2.5}$$

where we have defined,

$$R^2 = (A(\rho)\sinh\beta + \cosh\beta)^2 - \frac{h(\rho)\sinh^2\beta}{g_s^2 H(\rho)^2}, \tag{2.6}$$

$$a = \frac{1}{R^2}[A(\rho)\cosh 2\beta + (A(\rho)^2 + 1)\cosh\beta\sinh\beta - \frac{h(\rho)}{2g_s^2 H(\rho)^2}\sinh(2\beta)] \tag{2.7}$$

We have explicitly checked that the rotated background is a solution of the equations of motion. Notice that eq. (2.6) can be written as

$$R^2 = \cosh^2\beta + A(\rho)\sinh 2\beta + \left(\frac{R_T}{L_p}\right)^{7-p}\frac{\sinh^2\beta}{H(\rho)g_s^2} \tag{2.8}$$

which makes clear that R is strictly positive. In a similar way, eq. (2.7) can be written as

$$a = \frac{1}{HR^2}\left[\frac{1}{g_s}\alpha\cosh(2\beta) + \left(\frac{R_T}{L_p}\right)^{7-p}\frac{\sinh\beta\cosh\beta}{g_s^2} + H(\rho)\sinh\beta\cosh\beta\right]. \tag{2.9}$$

The metric (2.3) has the structure of a warped space,

$$ds^2 = \mathcal{H}(\rho)^{-1/2}\left[-h(\rho)dt^2 + d\vec{x}_p^2\right] + \mathcal{H}(\rho)^{1/2}\left[\frac{d\rho^2}{h(\rho)} + \rho^2 d\Omega_{8-p}\right], \tag{2.10}$$

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[2]In Appendix A, we will present the intermediate steps of this calculation for the extremal case $h = 1$.

where the new warp factor is

$$\mathcal{H}(\rho) = H(\rho)R(\rho)^2 = H(\rho)\cosh^2 \beta + \frac{\alpha}{g_s}\sinh 2\beta + (\frac{R_T}{L_p})^{7-p}\frac{\sinh^2 \beta}{g_s^2} \tag{2.11}$$

Note that $\mathcal{H}(\rho)$ is a harmonic function of the transverse space. Hence, after the rotation in p directions we are left with a Dp brane solution. Indeed, in terms of the new warp factor, the dilaton and gauge potential are

$$e^{2\phi[final]} = e^{2\phi(\infty)}\mathcal{H}(\rho)^{\frac{3-p}{2}}$$

$$\partial_\rho a(\rho) = \left[\frac{\alpha}{g_s} + (\frac{R_T}{L_p})^{7-p}\frac{\tanh \beta}{g_s^2}\right] \partial_\rho\left(\frac{1}{\mathcal{H}(\rho)}\right) = \tilde{\alpha}\ \partial_\rho\left(\frac{1}{\mathcal{H}(\rho)}\right), \tag{2.12}$$

which together with (2.10) define a Dp brane background with a β dependent RR charge. At infinity the warp factor asymptotes to a constant,

$$\mathcal{H}(\rho) \sim \cosh^2 \beta + \frac{\alpha}{g_s}\sinh 2\beta + (\frac{R_T}{L_p})^{7-p}\frac{\sinh^2 \beta}{g_s^2}, \tag{2.13}$$

and the space is asymptotically flat, as expected. However, it is interesting to note that even if we start with the Dp branes after the decoupling limit is taken, that is $H = \frac{L_p^{7-p}}{\rho^{7-p}}$, we will have in the UV that,

$$\mathcal{H}(\rho) \sim \frac{L_p^{7-p}}{\rho^{7-p}}\cosh^2 \beta + \frac{\alpha}{g_s}\sinh 2\beta + (\frac{R_T}{L_p})^{7-p}\frac{\sinh^2 \beta}{g_s^2} \tag{2.14}$$

So, again, the warp factor asymptotes to a constant. In other words, this rotation is taking the configuration *out of the decoupling limit* or coupling the field theory modes to gravity.

As is well known, before the rotation, the charge of the Dp brane solution is quantized. After the rotation the charge is,

$$\frac{1}{(2\pi l_s)^{7-p}} \int_{S^{8-p}} *F_{p+2} = \tilde{\alpha}\ \rho^{8-p}\mathcal{H}'(\rho)Vol_{S^{8-p}}$$

$$= \mathrm{Q}\cosh^2 \beta + R_T{}^{7-p}\frac{c_p}{2g_s}\sinh 2\beta \tag{2.15}$$

where $c_p = \frac{(7-p)Vol_{S^{8-p}}}{(2\pi l_s)^{7-p}g_s}$. Note that, generically, the charge is not quantized. This is not unusual since the supergravity dualities involved in the rotation procedure are a symmetry of the supergravity equations and not of the full string theory.

We will now move to study a more interesting example from the viewpoint of gauge-strings duality. We will apply the rotation procedure to the dual of a non-SUSY Yang-Mills-like theory, first presented in [17] and further studied in [18], [19].

2.1.1 Example: rotation of a dual to a Yang-Mills-like theory

The original background, consists of the decoupling limit of a stack of N_c D4 branes wrapping a circle with SUSY breaking boundary conditions. It was discussed with details in various publication, see for example [17], [18], [19]. We summarize it here (in string frame)[3]

$$ds^2 = H(\rho)^{-1/2}\Big[-h(\rho)dt^2 + d\vec{x}_{123}^2 + f(\rho)dx_4^2\Big] + H(\rho)^{1/2}\Big[\frac{d\rho^2}{f(\rho)h(\rho)} + \rho^2 d\Omega_4\Big],$$

$$e^{2\phi[initial]} = g_s^2 H^{-\frac{1}{2}},$$

$$F_6 = -\partial_\rho A(\rho)dt \wedge dx_1 \wedge .... \wedge dx_4 \wedge d\rho \quad F_4 = *F_6 \tag{2.16}$$

where $A(\rho) = \frac{1}{g_s H(\rho)}$ and Vol_4 is the volume of the unit four- sphere Ω_4.. The functions (in the low/zero temperature phase) are given by

$$H = (\frac{L}{\rho})^3, \quad f = 1 - (\frac{R_{kk}}{\rho})^3, \quad h = 1, \tag{2.17}$$

where $L^3 = N_c\pi g_s l_s^3$ and the coefficient R_{kk} is a free parameter. In the high temperature phase (with the coordinate x_4 compactified) we have,

$$H = (\frac{L}{\rho})^3, \quad h = 1 - (\frac{R_T}{\rho})^3, \quad f = 1 \tag{2.18}$$

where R_T is a free parameter related to the temperature of the system,

$$T = \frac{3}{4\pi}(\frac{L^3}{R_T})^{-1/2} \tag{2.19}$$

Proceeding as described in the previous section we rotate the background by applying the U-duality already discussed: four T-dualities in the worldvolume coordinates, uplift to M-theory, boost, reduce to IIA and T dualize back. The rotated background is,

$$ds^2 = \mathcal{H}(\rho)^{-1/2}\Big[-h(\rho)dt^2 + d\vec{x}_{123}^2 + f(\rho)dx_4^2\Big] + \mathcal{H}(\rho)^{1/2}\Big[\frac{d\rho^2}{f(\rho)h(\rho)} + \rho^2 d\Omega_4\Big],$$

$$e^{2\phi[final]} = g_s^2 \mathcal{H}^{-\frac{1}{2}},$$

$$F_6 = a'(\rho)dt \wedge dx_1 \wedge .... \wedge dx_4 \wedge d\rho \tag{2.20}$$

where the functions $\mathcal{H}(\rho), a(\rho)$ now read

$$\mathcal{H}(\rho) = H(\rho)\cosh^2\beta + \frac{1}{g_s}\sinh 2\beta + (\frac{R_T}{L})^3\frac{\sinh^2\beta}{g_s^2} \tag{2.21}$$

$$a(\rho) = \frac{1}{R(\rho)^2}\Big[A(\rho)\cosh 2\beta + (A(\rho)^2 + 1)\cosh\beta\sinh\beta - \frac{f(\rho)h(\rho)\sinh(2\beta)}{2g_s^2 H(\rho)^2}\Big] \tag{2.22}$$

---

[3] In [18] the authors performed a rescaling of the RR potential: $C_{p+1} \to \frac{\kappa_0^2\mu_{6-p}}{\pi}C_{p+1}$ and as a result the RR charge in their paper is measured in units of 2π. We do not perform such rescaling here.

and $R(\rho)^2 = \frac{\mathcal{H}(\rho)}{H(\rho)}$. As before, one can check explicitly that the background above is a solution of the eqs. of motion, but for this we must set either $R_T = 0$ for the low/zero temperature phase or set $R_{kk} = 0$, compactifying the x_4 direction, in the high temperature phase.

As explained in the previous section, in spite of having started with a decoupled background, after the rotation we have an asymptotically flat space, indicating that the rotation has coupled the field theory modes to the gravitational ones.

2.2   "Rotation" in p-q directions

We will study now a peculiar situation, where we have Dp branes and we separate a q manifold from the $p+1$ dimensional world-volume (we focus in the case $q = 2$ here, the reason for this will be explained in Appendix C). The internal manifold need not be specified. We will impose on our U-duality that:

- after the initial T-dualities, we are in IIA background, so that this is upliftable to M-theory

- that the boost generates a NS H_3 field, or what is equivalent, that the IIA configuration after the initial T-dualities contains an electric F_4.

We will assume (though this need not be the case) that the internal q-manifold is a torus. In that case, the solution reads (in string frame as usual)[4],

$$
\begin{aligned}
&ds^2 = H^{-1/2}[-dt^2 + dx_{p-q}^2 + d\sigma_q^2] + H^{1/2}[d\rho^2 + \rho^2 d\Omega_{8-p}^2], \\
&F_{p+2} = \partial_\rho \hat{A} dt \wedge dx_1 \wedge ... \wedge dx_{p-q} \wedge d\sigma_q \wedge d\rho, \\
&e^{2\phi[initial]} = e^{2\phi(\infty)} H^{\frac{3-p}{2}}.
\end{aligned}
\tag{2.23}
$$

We will proceed as follows: first we will do T dualities in the x_{p-q} directions, this will leave us in a IIA configuration, we will lift then to M-theory and boost. We reduce to IIA and T dualize back, all the details are written in Appendix C. We end up this duality-chain with a final configuration

$$
\begin{aligned}
&ds_{II,st}^2 = g_{tt}dt^2 + \frac{d\vec{x}_{p-2}^2}{H^{1/3}B^{1/2}} + B^{1/2}\left(H^{-2/3}d\vec{\sigma}_2^2 + H^{1/3}[d\rho^2 + \rho^2 d\Omega_{8-p}^2]\right), \\
&F_{p+2} = \partial_\rho \hat{A}(\cosh\beta + a_t \sinh\beta) \wedge d\rho \wedge d\sigma_1 \wedge d\sigma_2 \wedge dx_1 \wedge .... \wedge dx_{p-2}, \\
&H_3 = \sinh\beta \partial_\rho \hat{A} d\rho \wedge d\sigma_1 \wedge d\sigma_2, \\
&F_p = \partial_\rho(a_t) dt \wedge d\rho \wedge dx_1 \wedge .... \wedge dx_{p-2}, \\
&e^{2\phi[final]} = B^{\frac{5-p}{2}} H^{\frac{2-p}{3}}
\end{aligned}
\tag{2.24}
$$

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[4]we are dealing here with the Lorentz invariant case. Working with a non-extremal factor is straightforward.

where we have defined (See Appendix C for a detailed derivation),

$$A = H^{-2/3}[H\sinh^2\beta - \cosh^2\beta], \quad B = H^{-2/3}[H\cosh^2\beta - \sinh^2\beta],$$

$$C = 2H^{-2/3}\sinh\beta\cosh\beta(1-H), \quad g_{tt} = \frac{4AB - C^2}{4\sqrt{B}},$$

$$a_t = \frac{C}{2B} = \frac{\sinh\beta\cosh\beta(1-H)}{H\cosh^2\beta - \sinh^2\beta}, \tag{2.25}$$

Various aspects are worth emphasizing in this final background. Notice that:

- the metric is dual to a field theory with $SO(1, p-2)$ isometry, since

$$(4AB - C^2)H^{1/3} = -4 \rightarrow g_{tt} = -g_{x_i x_i} \tag{2.26}$$

- for $p = 5$ the dilaton is invariant under the whole U-duality procedure, since

$$e^{2\phi[initial]} = H^{\frac{3-p}{2}} = e^{2\phi[final]} = B^{\frac{5-p}{2}}H^{\frac{2-p}{3}} \tag{2.27}$$

- for $p = 5$ the procedure is generating charge of D3 brane (represented by the $F_{p=5}$) where the D3 branes are extended in the $p - q$ directions. This is the reason why the dilaton does not change as the D3 branes do not couple to it. The field H_3 is also generated.

In following sections, we will study a particular case of this U-duality, for a situation in which a set of D5 branes wrap a curved two-manifold (but we also have a black hole in the metric, breaking the $SO(1,3)$ isometry), the results are qualitatively the same. One should emphasize that there is yet another way of generating NS three-form fields, that is basically starting from NS five branes in IIA wrapping a three-cycle, see [20] for details.

Also, notice that we have imposed that after the first set of T-dualities, the background is solution of IIA Supergravity (the conditions for this to happen are discussed in Appendix C). Were this not the case, we present in Appendix D another possible solution generating technique with some applications.

Now, we will move to study an interesting application of the U-dualities discussed above.

3 Black Holes in $\mathcal{N} = 1$ SUSY Solutions

As anticipated, in this section we will apply the chain of dualities, lift and boost proposed in [6], to generate a new solution starting from a non-extremal solution in Type IIB. The interest of the original ('seed') solution is that it was argued to be dual to a field theory with minimal SUSY in four dimensions. The field theory was studied (at weak coupling) in [16] and is basically $\mathcal{N} = 1$ Super-Yang-Mills plus a sets of (massive) KK vector and chiral superfields

that UV complete the dynamics. This system was well studied and various string duals are known (particular solutions describing the field theory at strong coupling, with VEV's and certain operators deforming the Lagrangian). Let us briefly describe the general form of the string dual. The 'seed' background describes the backreaction of a set of N_c D5 branes wrapping a two-cycle inside the resolved conifold. It consists of a metric, dilaton $\phi(\rho)$ and RR three-form F_3 and, in string frame, it reads (the coordinates used are $[t, \vec{x}, \rho, \theta, \varphi, \tilde{\theta}, \tilde{\varphi}, \psi]$),

$$
\begin{aligned}
ds^2 &= e^{\phi(\rho)}\Big[dx_{1,3}^2 + e^{2k(\rho)}d\rho^2 + e^{2h(\rho)}(d\theta^2 + \sin^2\theta d\varphi^2) + \\
&+ \frac{e^{2g(\rho)}}{4}\left((\tilde{\omega}_1 + a(\rho)d\theta)^2 + (\tilde{\omega}_2 - a(\rho)\sin\theta d\varphi)^2\right) + \frac{e^{2k(\rho)}}{4}(\tilde{\omega}_3 + \cos\theta d\varphi)^2\Big], \\
F_{(3)} &= \frac{N_c}{4}\Big[-(\tilde{\omega}_1 + b(\rho)d\theta) \wedge (\tilde{\omega}_2 - b(\rho)\sin\theta d\varphi) \wedge (\tilde{\omega}_3 + \cos\theta d\varphi) + \\
&\quad b'd\rho \wedge (-d\theta \wedge \tilde{\omega}_1 + \sin\theta d\varphi \wedge \tilde{\omega}_2) + (1 - b(\rho)^2)\sin\theta d\theta \wedge d\varphi \wedge \tilde{\omega}_3\Big].
\end{aligned}
\tag{3.28}
$$

where $\tilde{\omega}_i$ are the left-invariant forms of $SU(2)$

$$
\begin{aligned}
\tilde{\omega}_1 &= \cos\psi d\tilde{\theta} + \sin\psi \sin\tilde{\theta}d\tilde{\varphi}, \quad \tilde{\omega}_2 = -\sin\psi d\tilde{\theta} + \cos\psi \sin\tilde{\theta}d\tilde{\varphi}, \\
\tilde{\omega}_3 &= d\psi + \cos\tilde{\theta}d\tilde{\varphi}.
\end{aligned}
\tag{3.29}
$$

This supersymmetric system was carefully studied in a series of papers [9], [10]; where it was shown that there is a combination of background functions (basically a 'change of basis') that move from a set of coupled BPS equations to a decoupled one that one can solve-up to one function, that we will call $P(\rho)$. We will not insist much with this formalism here and refer the interested reader to the original work [9], [10][5].

We will be more restrictive and for the purposes of this paper, we will study solutions where the functions $a(\rho) = b(\rho) = 0$ in the background of eq.(3.28). This is just in order to make our numerics simpler and illustrate the points we want to make. If we wanted to construct a black hole solution showing explicitly the transition between R-symmetry breaking and its restoration, we should then work without this restriction. Since in all known solutions the asymptotics of the functions $a(\rho) \sim b(\rho) \sim e^{-2\rho}$ our asymptotic results will be qualitatively correct, but near the black hole horizon there could be important differences.

We will then proceed as follows: first we will propose a background including a black hole with the restrictions mentioned above ($a(\rho) = b(\rho) = 0$). We will then pass it through the solution generating machine. This will produce a new background, now including also F_5, H_3, that we will write explicitly (we have checked that the equations of motion before and after the U-duality are the same). We will be explicit about the asymptotics of each of the functions

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[5]All this formalism was also applied to the case in which one also adds fundamental matter to the dual QFT or sources to the background above. We refer the interested reader to the review [21].

near the UV and near the horizon. This will produce for us a non-extremal generalization of the type IIB string background dual to the 'baryonic branch of KS-system' -certainly with the restriction that our background does not contain the information of the breaking of the R-symmetry from $Z_{2N} \to Z_2$.

3.1 A new 'seed' solution

In this section we consider the background presented in equation (3.28) with the restrictions $a = b = 0$ but including the non-extremality factors. In string frame we have,

$$ds^2_{IIB,s} = e^\phi \left[-h(\rho)dt^2 + dx_1^2 + dx_2^2 + dx_3^2 \right] + ds^2_{6,s},$$

$$ds^2_6 = e^\phi \left[\frac{e^{2k}}{s(\rho)} d\rho^2 + \frac{e^{2k}}{4} (\tilde\omega_3 + \cos\theta d\varphi)^2 + e^{2q}(d\theta^2 + \sin^2\theta d\varphi^2) + \frac{e^{2g}}{4}(d\tilde\theta^2 + \sin^2\tilde\theta d\tilde\varphi^2) \right],$$

$$F_{(3)} = \frac{N_c}{4} \left[-\tilde\omega_1 \wedge \tilde\omega_2 + \sin\theta d\theta \wedge d\varphi \right] \wedge (\tilde\omega_3 + \cos\theta d\varphi) = \frac{\alpha' N_c}{4} w_3, \tag{3.30}$$

where $h(\rho), s(\rho)$ are the non-extremality functions. In the following, we will set $s(\rho) = h(\rho)$ which is simply a choice of parametrization; this implies that (on a particular solution) other background functions $\{q(\rho), g(\rho), k(\rho), \phi(\rho)\}$ need not take the same values as in the SUSY background.

The above Ansatz might be familiar to some readers. It is worth highlighting a key difference with previous work. Since our goal is to 'rotate' this solution we look for solutions with stabilized dilaton. Namely, the typical asymptotic value of the dilaton inherited from the D5 (or NS5 brane) solution is linear [32], more precisely [23] [25],

$$e^\phi \sim e^\rho \rho^{-1/4} \tag{3.31}$$

We are looking for a dilaton that stabilizes at infinity, that is, which asymptotically behaves as:

$$e^\phi \sim 1 + \mathcal{O}\left(e^{-8/3\rho}\right). \tag{3.32}$$

Our solution is qualitatively characterized by two parameters: one describing the nonextremality of the solution and the other the speed at which the dilaton gets stabilized.

After applying the solution generating technique, we end up with a new Type IIB background containing F_5, H_3 aside from the fields originally present in eq.(3.30) (all details are

written in the Appendix E),

$$ds^2_{IIB,st} = H^{-1/2}\Big[-hdt^2 + dx_1^2 + dx_2^2 + dx_3^2 \Big] + e^{2\phi}H^{1/2}\Big[e^{2k}\frac{d\rho^2}{h(\rho)} + \frac{e^{2k}}{4}(\tilde{\omega}_3 + \cos\theta d\varphi)^2 +$$

$$+ e^{2q}(d\theta^2 + \sin^2\theta d\varphi^2) + \frac{e^{2g}}{4}(d\tilde\theta^2 + \sin^2\tilde\theta d\tilde\varphi^2)\Big],$$

$$H^{1/2} = \cosh\beta\sqrt{e^{-2\phi} - h(\rho)\tanh^2\beta}, \quad \phi_{new} = \phi(\rho),$$

$$F_3 = (\frac{\alpha' N_c}{4})\cosh\beta w_3, \quad H_3 = -\sqrt{h}(\frac{\alpha' N_c}{4})\sinh\beta\, e^{2\phi} *_6 w_3$$

$$F_5 = -\tanh\beta(1 + *_{10})\mathrm{vol}_4 \wedge d(\frac{h}{H}). \tag{3.33}$$

In the following, we will focus in the limit $\beta \to \infty$, this is the field theory-limit where the warp factors vanish at infinity. We will then rescale

$$\tilde{N} = N\cosh\beta, \quad x_i \to \sqrt{\cosh\beta}\sqrt{\tilde{N}\alpha'}\, x. \tag{3.34}$$

With all the above the $\beta \to \infty$ limits are finite and the solution is given by

$$ds^2_{IIB,st} = \tilde{N}\alpha'\Big[\tilde{H}^{-1/2}(-hdt^2 + dx_i dx^i) + e^{2\phi}\tilde{H}^{1/2}ds_6^2\Big],$$

$$F_3 = \frac{\alpha'\tilde{N}}{4}w_3, \quad H_3 = -\sqrt{h}\frac{\alpha'\tilde{N}}{4}e^{2\phi} *_6 w_3$$

$$F_5 = -(\alpha'\tilde{N})^2(1 + *_{10})\mathrm{vol}_4 \wedge d(\frac{h}{\tilde{H}}) \tag{3.35}$$

with

$$\tilde{H}^{1/2} = \sqrt{e^{-2\phi} - h(\rho)} \tag{3.36}$$

It is worth making a few observations:

- We have two new classes of solutions. The equations of motion before and after the 'rotation' are the same, thus a solution of the system of equations (3.30) furnishes a solution of the form (3.33)-(3.35).

- The non-extremality factor have made its way into the RR five form and the warp factor in eq.(3.36), but the dilaton and the three forms are unchanged from the extremal case. Notice that the factor of $\sqrt{h}$ in the NS field H_3 is canceled by the self dual in six dimensions $*_6 w_3$. See Appendix E for full details.

- The black hole in the seed solution will likely have negative specific heat but its dilaton is stabilized which is a crucial difference with solutions considered previously. We will discuss what is the behavior of the solution after the rotation.

In the following section we will explicitly describe the asymptotic behavior of the solution as well as its numerical presentation in the background given by eq.(3.30).

3.2 Asymptotics

We will proceed to study the asymptotics of the equations of motion and solve them numerically to obtain the new non-extremal seed solution (3.30).

3.2.1 UV expansion

A large ρ expansion of the functions (k, q, g, h, ϕ), that solve the equations of motion up to terms decaying faster than $e^{-4\rho}$ is [6]

$$h(\rho) = e^{-8x(\rho)} \sim 1 + C_2 e^{-\frac{8}{3}\rho},$$

$$e^{2q(\rho)} \sim \frac{N_c}{4}(2\rho - 1) + \frac{c}{4}e^{\frac{4}{3}\rho}, \quad e^{2g(\rho)} \sim \frac{N_c}{4}(1 - 2\rho) + \frac{c}{4}e^{\frac{4}{3}\rho},$$

$$e^{2k(\rho)} \sim \frac{2c}{3}e^{\frac{4}{3}\rho} - \frac{N_c^2}{6c}(25 - 40\rho + 16\rho^2)e^{-\frac{4}{3}\rho}, \quad e^{4\phi - 4\phi_0} \sim 1 + \frac{3N_c^2}{4c^2}(1 - 8\rho)e^{-\frac{8}{3}\rho}. \quad (3.37)$$

There are two parameters in the expansion above. The parameter C_2 characterizes the non-extremal behavior. The parameter c plays a quite important role. It is one of the integration constants of the BPS eqs. As an expansion in inverse powers of this constant it is possible to write a solution for the background in eq.(3.28)-see for example [10], [22]-such that when passed through this solution generating technique, it results in the Baryonic Branch dual solution of [7]. Physically (and in the background before the rotation), the constant c is describing the coupling of the dynamics on the five-brane to gravity and ultimately to the full string theory, it takes the configuration out of its 'near brane' limit and can be thought of as indicating the insertion of an irrelevant operator in the QFT lagrangian.

3.2.2 Near Horizon Asymptotics

Near the horizon, $\rho = \rho_h$, we expand the equations of motion in power series up to order 7. We have found,

$$e^{-8x(\rho)} = x_1(\rho - \rho_h) + x_2(\rho - \rho_h)^2 + x_3(\rho - \rho_h)^3 + \cdots + x_8(\rho - \rho_h)^8$$

$$e^{2q(\rho)} = q_0 + q_1(\rho - \rho_h) + q_2(\rho - \rho_h)^2 + \cdots + q_7(\rho - \rho_h)^7$$

$$e^{2g(\rho)} = g_0 + h_1(\rho - \rho_h) + g_2(\rho - \rho_h)^2 + \cdots + g_7(\rho - \rho_h)^7 \qquad (3.38)$$

$$e^{2k(\rho)} = k_0 + k_1(\rho - \rho_h) + k_2(\rho - \rho_h)^2 + \cdots + k_7(\rho - \rho_h)^7$$

$$e^{4\Phi(\rho)} = f_0 + f_1(\rho - \rho_h) + f_2(\rho - \rho_h)^2 + \cdots + f_7(\rho - \rho_h)^7$$

$$(3.39)$$

Demanding that the expansions in eq.(3.38) satisfy the equations of motion of the system we determine $x_2, ..., h_1..., g_1..., k_1..., f_1...$ in terms of x_1, h_0, g_0, k_0, f_0 and thus, there are only 5

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[6]Note that the UV expansion for $h(\rho) = e^{-8x}$ could have started with a lower order term such as $e^{-2\rho}$, see Section 4.1 for a discussion on the choice of parameters.
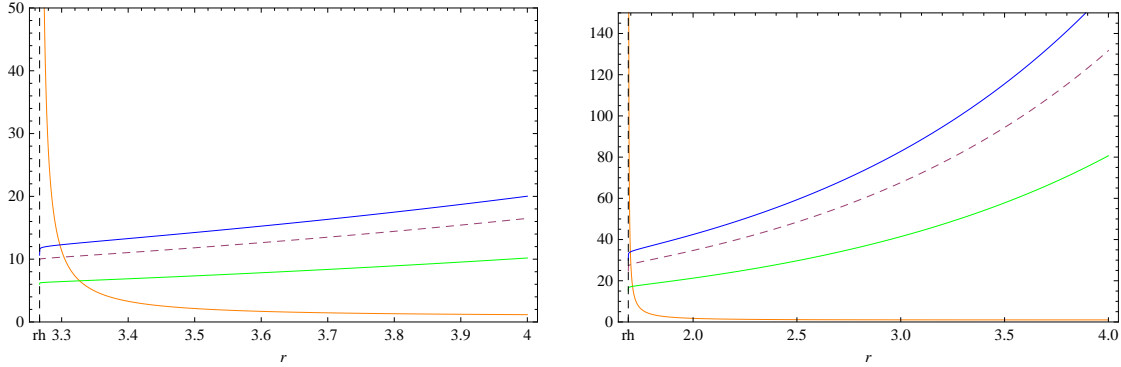
Figure 1: Solution before the rotation for $c = 1.97$, $C_2 = -7000$. The blue, dashed, green and orange lines represent e^q, e^k, e^g and h^{-1} respectively.

Figure 2: Solution before the rotation for $c = 125.62$, $C_2 = -89$. The blue, dashed, green and orange lines represent e^q, e^k, e^g and h^{-1} respectively.

independent parameters coming from the expansion at the horizon. Furthermore, x_1 is related to the non-extremality parameter α-see Appendix F for the definitions,

$$\alpha = -\frac{x_1 \sqrt{f_0} g_0 h_0}{8} \tag{3.40}$$

3.3 Numerics

The strategy we will follow is to numerically integrate back from infinity the equations of motion in Appendix F, using as boundary conditions the expansion in the UV-eq.(3.37)-and require that at the horizon this numerical solution matches the expansion (3.38) and its first derivatives -up to seventh order. Doing this for a given set of values (c, C_2), determines the free parameters $(x_1, h_0, g_0, k_0, f_0)$. Note that not for every pair of (c, C_2) there is a black hole solution. For some values of (c, C_2) there will not be a horizon and for others there might be naked singularities outside the horizon. The study of the this two-parameter family of solutions for the full (c, C_2) parameter space is an interesting question that we will not address here. We concentrate on finding particular values of (c, C_2) that will produce black hole solutions. Needless to say, finding a solution before the rotation guarantees that we will have a solution after the rotation. Examples of numerical solutions are shown in Figure 1 and 2 and the respective dilatons in Figures 3 and 4.

3.3.1 Comments about the numerical method

As mentioned above we integrate back from infinity and match with the expansion at the horizon. In order to do so we define a "mismatch" function evaluated at a point ρ_0 close to
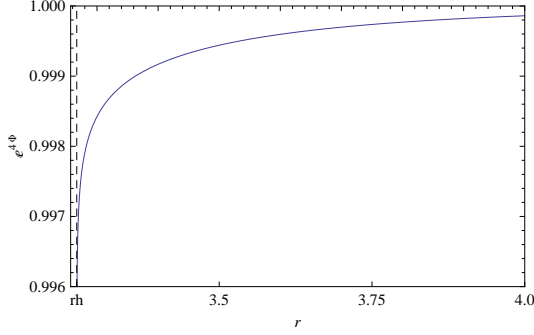
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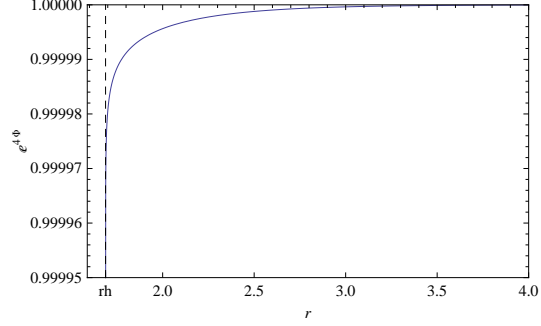Figure 3: Dilaton, $e^{4\Phi}$ for $c = 1.97, C_2 = -7000$.

Figure 4: Dilaton, $e^{4\Phi}$ for $c = 125.62, C_2 = -89$.

the horizon, $\rho_0 = \rho_h + 0.15$

$$m(\rho_0) = [g_{sh}(\rho_0) - g_{num}(\rho_0)]^2 + [g'_{sh}(\rho_0) - g'_{num}(\rho_0)]^2 + [h_{sh}(\rho_0) - h_{num}(\rho_0)]^2 +$$
$$[h'_{sh}(\rho_0) - h'_{num}(\rho_0)]^2 + [k_{sh}(\rho_0) - k_{num}(\rho_0)]^2 + [k'_{sh}(\rho_0) - k'_{num}(\rho_0)]^2 +$$
$$[\Phi_{sh}(\rho_0) - \Phi_{num}(\rho_0)]^2 + [\Phi'_{sh}(\rho_0) - \Phi'_{num}(\rho_0)]^2. \tag{3.41}$$

The subscript num denotes the numerical solution obtained by integrating back from infinity and sh denotes the series expansion (3.38) -evaluated at ρ_0. Recall that the expansion at the horizon depends on five free parameters, $(x_1, h_0, g_0, k_0, f_0)$. For a given set of (c, C_2) we minimize the mismatch function to determine $(x_1, h_0, g_0, k_0, f_0)$. The parameters at infinity (c, C_2) are adjusted so that $m(\rho_0) < 5 \times 10^{-4}$. All the numerical procedures where done in Mathematica using WorkingPrecission 40. We also checked that the constraint $T + U = 0$ (see appendix F) remains numerically negligible throughout the domain.

3.4 The rotated solution

Once we have obtained a numerical solution for the non-extremal background (3.30) we can easily generate the rotated solution as summarized in eq.(3.35) and outlined in the Appendix E. Before presenting the numerical results let us look at the large ρ asymptotics. Using the UV expansions of the seed solution (3.37), one can obtain the black hole metric (3.35) asymptotics

after the rotation,

$$-g_{tt} \sim \frac{e^{4/3\rho}}{A(\rho)} - \frac{e^{-4/3\rho}}{512c^4\sqrt{2}A(\rho)^3}[256c^4C_2^2 + 27(1 - 8\rho)^2 + 96c^2C_2(-1 + 8\rho)] + \cdots ,$$

$$g_{xx} \sim \frac{e^{4/3\rho}}{A(\rho)} - \frac{e^{-4/3\rho}}{512c^4\sqrt{2}A(\rho)^3}[27(1 - 8\rho)^2] + \cdots ,$$

$$g_{\rho\rho} \sim \frac{2c}{3}A(\rho) + \cdots , \quad g_{\theta\theta} \sim \frac{c}{4}A(\rho) + \frac{e^{-4/3\rho}}{4}A(\rho)(2\rho - 1) + \cdots ,$$

$$g_{\psi\psi} \sim \frac{c}{6}A(\rho) + \cdots , \quad g_{\tilde{\theta}\tilde{\theta}} \sim \frac{c}{4}A(\rho) - \frac{e^{-4/3\rho}}{4}A(\rho)(2\rho - 1) + \cdots . \tag{3.42}$$

Where we have defined the quantity

$$c\sqrt{8}A(\rho) \equiv \sqrt{24\rho - 8C_2c^2 - 3}. \tag{3.43}$$

Using a variable $u = e^{2\rho/3}$ the metric can be written -to leading order- as,

$$ds^2 = \frac{u^2}{A(u)}(dx_i dx^i) + \frac{3cA(u)}{2}(\frac{du^2}{u^2} + ds_{T^{11}}^2) + \cdots \mathcal{O}(u^{-2}). \tag{3.44}$$

Thus, the non-extremal solution found after the rotation has the asymptotics of the Klebanov-Strassler background [8]. We point out that the reason we get precisely Klebanov-Tseytlin type of asymptotics lies in our precise arrangements of coefficients in the seed solution. Namely we arranged for a seed solution with no linear dilaton and with a dilaton behavior that, in the field theory limit, eliminates the leading term in the harmonic function given in equation (3.36).

With our careful choice of asymptotic conditions for the seed solution of D5 branes we have managed to construct black holes in asymptotically KT backgrounds. Our solution is different from those obtained in the traditional approach to black holes in asymptotically KT backgrounds (see, for example, [33]). One glaring difference is the form of the RR fluxes. In particular the presence of the non-extremality parameter in F_5 and in the warp factor $H(\rho)$ is completely novel.

In Figures 5-8 we present the numerical plots of the diffent metric elements after the rotation for two sets of parameters. In Figures 9 and 10 we see how the angular part of the solution approaches the $T^{1,1}$ metric for the two representative solutions.

3.5   Comments on thermodynamics

Let us clarify the expectations of the form of the background that arise from field theory considerations. In the previous section we obtained a solution largely characterized by two parameters (c, C_2). After the rotation, the interpretation of c has been spelled out explicitly
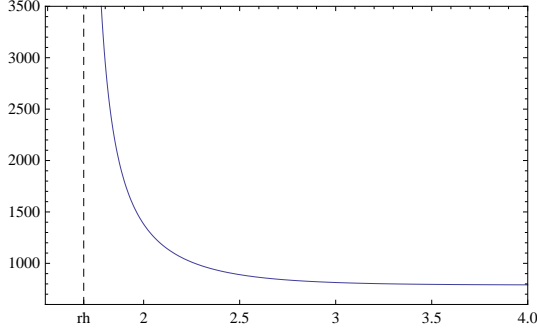
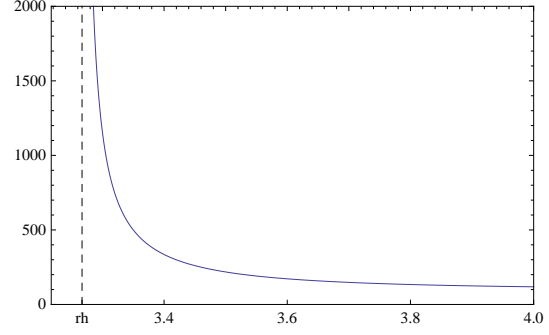Figure 5: The g_{rr} metric element for a so-lution after the rotation, $c = 125.62$, $C2 = -89$.

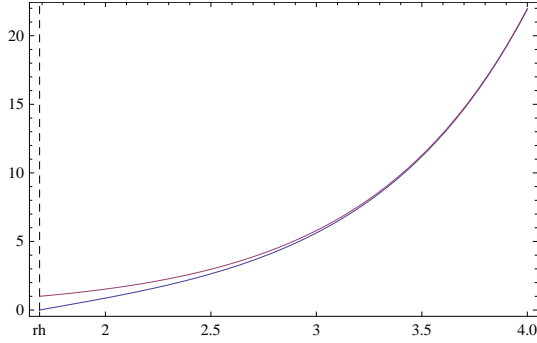Figure 6: The g_{rr} metric element for a solu-tion after the rotation, $c = 1.97$, $C_2 = -7000$.

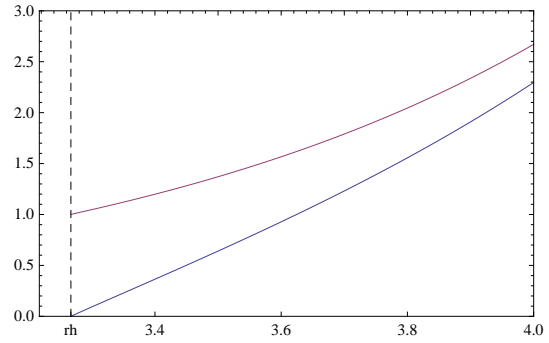Figure 7: g_{xx} and g_{tt} for a solution after the rotation, $c = 125.62$, $C_2 = -89$.

Figure 8: g_{xx} and g_{tt} for a solution after the rotation, $c = 1.97$, $C_2 = -7000$.

Figure 9: Angular part of the metric after the rotation. We plot $g_{\theta\theta}$, $g_{\tilde{\theta}\tilde{\theta}}$ and $\frac{3}{2}g_{\psi\psi}$. So-lution with $c = 125, C_2 = -89$.
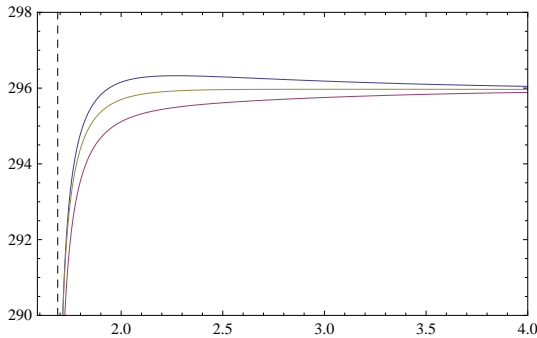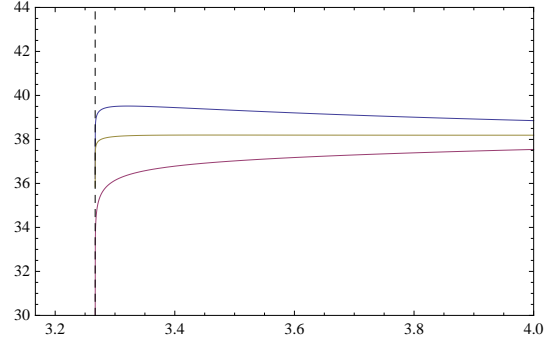

Figure 10: Angular part of the metric after the rotation. We plot $g_{\theta\theta}$, $g_{\tilde{\theta}\tilde{\theta}}$ and $\frac{3}{2}g_{\psi\psi}$. So-lution with $c = 1.97, C_2 = -7000$.

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in [15], [6] and [13]. It is related to the vev of the baryon operator. The other parameter present in the solution is C_2, its interpretation coincides roughly with the temperature of the solution in the field theory.

We will present and exhaustive analysis of these classes of solutions elsewhere and will limit our attention here to a few main points. The main deterrent in presenting a final description of the thermodynamics is the need to explore a large parameter space of the solutions as well as the difficulty in matching data at asymptotic infinity with data near the horizon. The analysis is numerically costly but we emphasize that these are not difficulties of principle. Our goal in this work is to explicitly show, through numerical analysis, the existence of the conjectured solutions.

Let us show that the thermodynamic analysis can, in principle, be done explicitly for the solutions we have. For the temperature, we use the standard approach of looking at the Euclidean section and imposing the absence of conical singularities to determine the temperature of the supergravity backgrounds. Basically, for a metric with Euclidean section of the form

$$ds^2 = f d\tau^2 + \frac{d\rho^2}{g} + \dots, \tag{3.45}$$

we find a temperature equal to

$$T = \frac{1}{4\pi} \sqrt{f' g'}, \tag{3.46}$$

where prime represents derivative with respect to the radial coordinate and the above expression is evaluated at the horizon. Interestingly, the temperature in both backgrounds, that is, before and after the rotation, is the same. In terms of the near horizon data we have

$$T = \frac{1}{4\pi} \frac{x_1}{\sqrt{k_0}}. \tag{3.47}$$

It is worth noting that this is the temperature at the horizon. In the case of asymptotically flat backgrounds (before the rotation) we have the option of computing the temperature as seen by an observer at infinity. We refer the reader to [23] for a detailed discussion of this procedure in the D5 or NS5 background. The entropy is computed, as usual, as the area of the horizon. The most complicated quantity to compute is the free energy which is required to compute, for example, the specific heat. For the free energy we need to evaluate the action. In the case of asymptotically flat solutions for D5/NS5 branes we can essentially follow [23]. A very preliminary analysis seems to confirm our intuition about the specific heat of such solutions. Namely, evaluation of the action suggests that before the rotation the solutions obtained have negative specific heat. A precise evaluation of the action after the rotation is more subtle as naive evaluation yields divergent terms. It is likely that a more accurate approach requires some sort of holographic renormalization or substraction along the lines of [33]. Preliminary evaluation suggests a positive specific heat for the rotated solutions [7].

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[7] Dario Martelli suggested that along the lines of the first two papers in [3], some thermodynamical quantities should be invariant under T-dualities, shifts of coordinates and boosts. See also [34].

4 About decoupling limits

It is worth discussing a bit about decoupling limits. Let us do this in the extremal case, the extension to the cases with non extremal factors follows from what we write here. The first point to emphasize is that when gravity modes are coupled to a generic D-brane configuration, the warp factors asymptote to a constant. An example of this is what happens to the flat Dp branes, with $H = 1 + (\frac{Q}{\rho})^{7-p}$.

We will compare warp factors in the examples studied in the previous sections. It is enough to focus on the expressions of the metric component g_{tt}. In all the examples studied we found

$$g_{tt} = \frac{4AB - C^2}{4\sqrt{B}}. \tag{4.48}$$

Indeed, in the case in which we rotate a flat Dp-brane, we have eq.(A.6) with A, B, C given in eq.(A.5). In the case of the rotation in $p - q$ directions the results for A, B, C are written in eq.(2.25) and in the case of the wrapped D5's are given in eq.(E.9). Again, for the point we wish to emphasize, we will focus on the extremal cases, when the rotation preserves $SO(1, p)$. So, in these cases we have[8],

$$g_{tt,(p-q)} = -\frac{1}{\cosh \beta \sqrt{H - \tanh^2 \beta}}, \; g_{tt,p} = -\frac{\sqrt{H}}{\sqrt{H^2(\hat{A} \tanh \beta - 1)^2 - \tanh^2 \beta}},$$

$$g_{tt,D5} = -\frac{1}{\cosh \beta \sqrt{e^{-2\phi} - \tanh^2 \beta}} \tag{4.49}$$

The case of the rotation in p directions shows that the induced charge of D0 brane (after the first set of T-dualities) as shown by eq.(A.2), enters in the new warp factor via the one-form potential $A_1 = \hat{A}dt$. This makes the decoupling of the final configuration more unclear and typically, if we start with 'decoupled' Dp branes (that is $H \sim \rho^{p-7}$) we will generate a non-decoupled configuration, with an asymptotically constant warp factor as discussed around eq.(2.13). Of course, we can try to take the decoupling limit in the final configuration.

On the other hand the cases of the wrapped D5's and the rotation in $p - q$ directions (that share the fact that a D0 charge is induced by the boost only) are such that we *must* start with a non-decoupled configuration in order to avoid a generated background with a singularity or end of the space in the UV (zero warp factor at some finite value of the radial coordinate). Indeed, if $H = 1 + (\frac{Q}{\rho})^{7-p}$ we choose a particular value of the boost parameter $\beta \to \infty$, together with a particular choice of the value of $e^{\phi(\infty)} = 1$ in the D5's case) to obtain a background decoupled from gravity[9].

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[8]Gregory Giecold correctly pointed out that in the non-extremal cases the non-extremality factor $h(\rho)$ enters as expected, for example $g_{tt,D5} = -\frac{h}{\cosh \beta \sqrt{e^{-2\phi} - h \tanh^2 \beta}}$. We thank him for this and various other correct comments.

[9] One of the interesting observations in [6], is that in the case of the D5's the solution described in the

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4.1   Choice of Parameters for $\mathcal{N} = 1$ Black Holes

Let us discuss in more detail the choice of parameters in the solution. This is a particularly sensitive question from the numerical and from the physical points of view. We note that *a priori* there are ten parameters in our solution as corresponds to a system of five second order ordinary differential equations. The constraint lowers this number by one. At this point the simplest route for us is to notice that we are looking for a particular solution that asymptotes to the extremal supersymmetric one found in [10],[13] (see, in particular, appendix B there). In that context most of our constants get fixed, as we require the same UV behavior; we use the expressions in that solution to fix six constants. Of the three constants left, one gets fixed by the asymptotic behavior of the non-extremality function, that is, we choose

$$h(\rho) \sim 1 - C_2 e^{-\frac{8}{3}\rho}. \tag{4.50}$$

The leading coefficient guarantees that we will have, after rotation, a solution with KT asymptotics. The key observation to be made is that, in principle, there is a mode allowed that we have explicitly set to zero. Namely, another solution to the equations of motion in the asymptotic regime could be

$$h(\rho) \sim 1 + C_1 e^{-2\rho} - C_2 e^{-\frac{8}{3}\rho} + \dots. \tag{4.51}$$

The above expression should be thought of schematically as the solution to a second order differential equation for $h(\rho)$ which is defined asymptotically by two parameters. This C_1 is one of the three constants that we fix and therefore we remain with two constants that we called (c, C_2). The parameter c has the same meaning discussed in [10],[13] and C_2 is related to the temperature. Thus, the condition that the solution after the rotation has KT asymptotics plus a condition on the non-extremality function determine the values of the integration constants.

It is worth noting that selecting the right constants in the D3/D5 picture is more cumbersome as there are more parameters in the generic D3/D5 approach to black holes with KT asymptotics. For example, our solution does not allow for a generic mode that will lead to a Freund-Rubin form for F_5. As has been repeatedly emphasized, our F_5 is related to the non-extremality function and deforms the typical term proportional to the volume 5-form. Let us explicitly visualize the difficulty in choosing the asymptotic expansion from the D3/D5 point of view in another example. The choice $C_1 = 0$ roughly corresponds to a dimension 3 operator in the D3/D5 picture. This can be seen easily by going to the radial coordinate $u = \exp(2\rho/3)$.

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papers [10], [22] is such that the dilaton asymptotes to a (tunable) constant and the subleading terms are precisely those needed to generate the warp factor of Klebanov-Strassler

5 Conclusions and Future Directions

We present here some concluding remarks together with some possible future projects to complement and refine the material in this paper.

In this work we have elaborated on the U-duality proposed in [6]. In that case, it was used to connect solutions describing wrapped D5 branes with the family of solutions describing the strong coupling dynamics of the baryonic branch of the Klebanov-Strassler field theory. In our case, we have used this U-duality to construct a new family of solutions describing the finite temperature phase of this baryonic branch. But before doing so, we explored the U-duality (that, following custom, we called rotation) in more generality, as a solution generating technique. We applied it to flat Dp branes in two different circumstances so that we appreciated the subtle differences making the choice of dualities in [6] so special.

Once we developed this technology, we applied it to rotate the Witten model for a Yang-Mills dual [17]. All this represents new material and it would be interesting to find an application for it, hopefully in the context of duals to Yang-Mills or QCD-like field theories.

In section 3, we discussed various new things: first a class of non-extremal solutions describing five branes wrapping a two-cycle inside the resolved conifold. The main feature of these non-extremal backgrounds is that the dilaton is asymptotically stabilized, which we interpreted as coupling the dual field theory to gravity modes. Then, we applied the U-duality to this solution, to find a new background of IIB that describes a non-extremal deformation of the Klebanov-Tseytlin solution (*decoupled* from gravity or stringy modes). We briefly commented on the associated thermodynamic properties of this new family of backgrounds. Finally, we discussed various aspects of this coupling/decoupling from gravity modes in section 4.

Let us now list a set of projects that complement the material presented here and that may improve the understanding of the topic:

- It is important to sort out the details of the thermodynamics *after* the rotation in the solutions presented in section 3. Indeed, as we indicated in section 3.5, this poses a numerical problem, but certainly does not amount to a conceptual obstruction. The result for quantities like the specific heat or the free energy will be of great interest. Notice that this presents a novel way to approach the finite temperature phase of the Klebanov-Strassler field theory. Comparison with the solutions of [38] would be interesting.

- More conceptually; it would be nice to better understand the role played by the D0 branes in this solution generating technique. Indeed, notice the differences between the examples analyzed in section 2 and the one in section 2.1, all due to the presence of D0 branes *before* the boost that translates into the different final results. In the same vein, it would be good to have a better understanding on the reasons for the equivalence between the U-duality explained here and the rotation of the J_2, Ω_3 forms as described

in [12], [13].

- It would be of very much interest to extend our non-extremal solution to a background of the form in eq.(3.28) where the restriction $a(\rho) = b(\rho) = 0$ is not applied. Indeed, finding such a non-extremal solution would, after the application of the U-duality, produce a solution dual to a non-SUSY KS-like field theory in the baryonic branch. It may prove interesting to compare this new solution with that presented in the papers [35]. The different nature of the non-SUSY deformation will certainly reflect in different field theory aspects.

- It would be nice to use the solution presented in section 3 as a base to produce a non-SUSY deformation along the line proposed recently in [36]. The authors of [36] are finding SUSY breaking backgrounds by adding anti-D3 branes. Their solutions have singularities (and the problem is if such a singularity is physically acceptable). Proceeding as with the solution in this paper may set us on a different branch of solutions.

- It would be interesting to find a Physics application of the new backgrounds of section 2.1.1.

- It would be nice to extend the treatment here to the case in which the seed background contains $N_f \sim N_c$ flavors. This would be following the lead of the papers [24], [25]. The subtle point would be how to rotate such solutions (in other words, how the sources get represented in M-theory, see [37]. Comparison with the solutions of [38] would be interesting)

- We have presented two numerical solutions as examples of the type of backgrounds obtained. These examples correspond to specific values of the UV parameteers. We did not attempt a study of the full family of solutions in the whole parameter space. This is an interesting question worth investigating.

We hope to address some of these problems in the near future.

Acknowledgments:

A   Appendix: U-duality in the case of flat Dp-branes

In this appendix we will give a sketchy derivation of the U-duality chain, in the case in which we rotate flat Dp-branes. We will not consider the non-extremal case, this can be done easily and the result of doing so is included in the first section of the paper. Here we want just to emphasize some points.

We start from the SUSY configuration in type II supergravity (string frame),

$$ds^2 = H^{-1/2}(-dt^2 + dx_p^2) + H^{1/2}(d\rho^2 + \rho^2 d\Omega_{8-p}),$$

$$e^{2\phi} = H^{\frac{3-p}{2}}, \ F_{p+2} = \partial_\rho \hat{A} dt \wedge dx_1 \wedge ...dx_p \wedge d\rho \tag{A.1}$$

where $\hat{A} = -A$ in the notation of Section 2. Following standard convention we define the charge of F_{8-p} as positive. The sign of F_{p+2} will then depend on whether we consider Minkowski or Euclidean worldvolume on the D_p brane. We apply T-duality in the x_p directions,

$$ds^2 = -H^{-1/2}dt^2 + H^{1/2}(dx_p^2 + d\rho^2 + \rho^2 d\Omega_{8-p}),$$

$$e^{2\phi} = H^{\frac{3}{2}}, \ F_2 = \partial_\rho \hat{A} dt \wedge d\rho \tag{A.2}$$

We lift this to M-theory, using $e^{\frac{4\phi}{3}} = H$,

$$ds_{11}^2 = H(dx_{11} + \hat{A}dt)^2 - H^{-1}dt^2 + dM_9, \ \ dM_9 = (dx_p^2 + d\rho^2 + \rho^2 d\Omega_{8-p}) \tag{A.3}$$

we now boost in the $t - x_{11}$ direction with rapidity β

$$dt \to \cosh\beta dt - \sinh\beta dx_{11}, \ \ dx_{11} \to -\sinh\beta dt + \cosh\beta dx_{11}, \tag{A.4}$$

to get

$$ds_{11}^2 = Adt^2 + Bdx_{11}^2 + Cdtdx_{11} + dM_9,$$

$$A = H^{-1}[H^2(\hat{A}\cosh\beta - \sinh\beta)^2 - \cosh^2\beta]$$

$$B = H^{-1}[H^2(\hat{A}\sinh\beta - \cosh\beta)^2 - \sinh^2\beta]$$

$$C = -2H^{-1}[H^2(\hat{A}\cosh\beta - \sinh\beta)(\hat{A}\sinh\beta - \cosh\beta) - \cosh\beta\sinh\beta] \tag{A.5}$$

and we prepare the metric for reduction back to IIA as

$$ds_{11}^2 = B(dx_{11} - a_t dt)^2 + B^{-1/2}\left[g_{tt}dt^2 + B^{1/2}dM_9\right],$$

$$a_t = -\frac{C}{2B}, \ \ g_{tt} = \frac{4AB - C^2}{4\sqrt{B}} \tag{A.6}$$

we now write the IIA configuration as,

$$ds^2_{IIA} = g_{tt}dt^2 + B^{1/2}(dx_p^2 + d\rho^2 + \rho^2 d\Omega_{8-p}), \ e^{2\phi} = B^{3/2}, \ F_2 = \partial_\rho a_t dt \wedge d\rho \quad \text{(A.7)}$$

and we T-dualize back in the x_p directions to get

$$ds^2_{II} = g_{tt}dt^2 + B^{-1/2}dx_p^2 + B^{1/2}(d\rho^2 + \rho^2 d\Omega_{8-p}),$$
$$e^{2\phi} = B^{\frac{3-p}{2}}, \quad F_{p+2} = \partial_\rho a_t dt \wedge d\rho \wedge dx_1.... \wedge dx_p \quad \text{(A.8)}$$

We see that the $SO(1,p)$ invariance is preserved, as $g_{tt}\sqrt{B} = -1$.

B   Appendix: Black Dp branes

In this appendix we gather some useful facts about black Dp-brane solutions [26],[27],[28]. The metric in string frame is,

$$ds^2 = -\frac{f_+(r)}{\sqrt{f_-(r)}}dt^2 + \sqrt{f_-(r)}\sum_{i=1}^{p} dx^i dx^i + \frac{f_-(r)^{-\frac{1}{2}-\frac{5-p}{7-p}}}{f_+(r)}dr^2 + r^2 f_-(r)^{\frac{1}{2}-\frac{5-p}{7-p}}d\Omega^2_{8-p} \quad \text{(B.1)}$$

where

$$f_\pm(r) = 1 - \left(\frac{r_\pm}{r}\right)^{7-p} \quad \text{(B.2)}$$

The mass per unit volume M and the R-R charge N are

$$M = \frac{1}{(7-p)(2\pi)^7 d_p l_P^8}((8-p)r_+^{7-p} - r_-^{7-p}), \quad N = c_p(r_+ r_-)^{\frac{7-p}{2}}, \quad \text{(B.3)}$$

where l_P is the ten dimensional Planck length $l_P = g_s^{1/4}l_s$ and $c_p, \ d_p$ are numerical factors,

$$d_p = 2^{5-p}\pi^{\frac{5-p}{2}}\Gamma\left(\frac{7-p}{2}\right)$$

$$c_p = \frac{1}{d_p g_s l_s^{7-p}} = \frac{(7-p)Vol_{S^{8-p}}}{(2\pi l_s)^{7-p}g_s} \quad \text{(B.4)}$$

The background also has a nontrivial dilaton and a R-R flux,

$$e^{-2\Phi} = g_s^{-2}f_-(r)^{-\frac{p-3}{2}}$$

$$\frac{1}{(2\pi l_s)^{7-p}}\int_{S^{8-p}} *F_{p+2} = N. \quad \text{(B.5)}$$

The Einstein frame metric ($g_{E\mu\nu} = \sqrt{g_s e^{-\Phi}}g_{st\mu\nu}$) has a horizon at $r = r_+$ and, for $p \leq 6$, a singularity at $r = r_-$. If $r_+ > r_-$ the singularity is covered by the horizon and the solution is a black hole. In the extremal case, $r_+ = r_-$, the space is singular except for $p = 3$.

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Let us define a new coordinate,

$$\rho^{7-p} = r^{7-p} - r_-^{7-p}.$$

This change of variables transforms (B.1) to a more familiar form,

$$ds^2 = H(\rho)^{-\frac{1}{2}} \left(-h(\rho)dt^2 + \sum_{i=1}^{p} dx^i dx^i \right) + H(\rho)^{\frac{1}{2}} \left(\frac{d\rho^2}{h(\rho)} + \rho^2 d\Omega_{8-p}^2 \right) \tag{B.6}$$

where

$$H(\rho) = 1 + \frac{r_-^{7-p}}{\rho^{7-p}}$$

$$h(\rho) = 1 - \frac{r_+^{7-p} - r_-^{7-p}}{\rho^{7-p}} \tag{B.7}$$

Throughout the paper we use the notation,

$$L_p^{7-p} \equiv r_-^{7-p} \quad \text{and} \quad R_T^{7-p} \equiv r_+^{7-p} - r_-^{7-p}.$$

Note that using (B.3) one can also write

$$L_p^{7-p} = \tilde{Q} \left[\sqrt{1 + \left(\frac{R_T^{7-p}}{2\tilde{Q}} \right)^2} - \frac{R_T^{7-p}}{2\tilde{Q}} \right]$$

$$= \tilde{Q} \, \alpha_p. \tag{B.8}$$

where $\tilde{Q} = \frac{N}{c_p}$ and $\alpha_p = (1 + (\frac{R_T^{7-p}}{2\tilde{Q}})^2)^{1/2} - \frac{R_T^{7-p}}{2\tilde{Q}}$.

¿From (B.3) we can see that the parameter R_T is related to the energy per unit volume above extremality ϵ,

$$\Delta M = \epsilon = \frac{r_+^{7-p} - r_-^{7-p}}{(2\pi)^7 d_p l_P^8} = \frac{R_T^{7-p}}{(2\pi)^7 d_p l_P^8}. \tag{B.9}$$

Note that in the decoupling limit ($\alpha' \to 0$, energies fixed) ϵ remains fixed and $\alpha_p \to 1$.

In the extremal case, $r_+ = r_-$, we have $R_T = 0$ and $L_p^{7-p} = \tilde{Q} = N d_p g_s l_s^{7-p}$.

C   Appendix: Some details on the U-duality

Here we provide details for the derivations of eq.(2.24). We started with

$$ds^2 = H^{-1/2}[-dt^2 + dx_{p-q}^2 + d\sigma_q^2] + H^{1/2}[d\rho^2 + \rho^2 d\Omega_{8-p}^2],$$

$$F_{p+2} = \partial_\rho \hat{A} dt \wedge dx_1 \wedge ... \wedge dx_{p-q} \wedge d\sigma_q \wedge d\rho,$$

$$e^{2\phi[initial]} = e^{2\phi(\infty)} H^{\frac{3-p}{2}}. \tag{C.1}$$

The we perform the U-duality described in the text below eq.(2.23)

$$ds^2 = H^{-1/2}[-dt^2 + d\sigma_q^2] + H^{1/2}[d\rho^2 + \rho^2 d\Omega_{8-p}^2 + dx_{p-q}^2],$$
$$F_{q+2} = \partial_r \hat{A} dt \wedge d\sigma_q \wedge d\rho,$$
$$e^{2\phi} = e^{2\phi(\infty)} H^{\frac{3-q}{2}}. \tag{C.2}$$

Now, we need this to be a configuration in Type IIA (in order to lift this to M-theory). We will also want to impose that when lifted to M-theory this will produce a four-form field G_4. If this is the case, we must have that either $q = 2$ or that $q = 4$ (the cases of $q = 0, q = 6$ are analogous to what we analyzed in the first section). The case $q = 2$, on which we will elaborate upon below, has the peculiarity that the reduction from eleven dimensions back to IIA will generate a NS H_3 field, proportional to the boost. Let us see this in detail. To begin with, we will lift to eleven dimensions the configuration in eq.(C.2).

$$ds_{11}^2 = H^{1/3} dx_{11}^2 + H^{-2/3}[-dt^2 + d\vec{\sigma}_2^2] + H^{1/3}[d\rho^2 + \rho^2 d\Omega_{8-p}^2 + d\vec{x}_{p-2}^2],$$
$$G_4 = \partial_\rho \hat{A} dt \wedge d\rho \wedge d\sigma_1 \wedge d\sigma_2 \tag{C.3}$$

and now we boost with rapidity β

$$dt \to \cosh\beta dt - \sinh\beta dx_{11}, \quad dx_{11} \to -\sinh\beta dt + \cosh\beta dx_{11} \tag{C.4}$$

and we can rewrite the configuration after the boost as,

$$ds_{11}^2 = Adt^2 + Bdx_{11}^2 + Cdtdx_{11} + dM_9^2,$$
$$G_4 = \partial_\rho \hat{A}(\cosh\beta dt - \sinh\beta dx_{11}) \wedge d\rho \wedge d\sigma_1 \wedge d\sigma_2,$$
$$dM_9^2 = H^{-2/3} d\vec{\sigma}_2^2 + H^{1/3}[d\rho^2 + \rho^2 d\Omega_{8-p}^2 + d\vec{x}_{p-2}^2],$$
$$A = H^{-2/3}[H\sinh^2\beta - \cosh^2\beta],$$
$$B = H^{-2/3}[H\cosh^2\beta - \sinh^2\beta],$$
$$C = 2H^{-2/3}\sinh\beta\cosh\beta(1 - H) \tag{C.5}$$

when we reduce this to IIA we get,

$$ds_{IIA,st}^2 = g_{tt}dt^2 + B^{1/2}\left(H^{-2/3}d\vec{\sigma}_2^2 + H^{1/3}[d\rho^2 + \rho^2 d\Omega_{8-p}^2 + d\vec{x}_{p-2}^2]\right),$$
$$e^{2\phi[A]} = B^{3/2}, \quad g_{tt} = \frac{4AB - C^2}{4\sqrt{B}},$$
$$F_4 = \partial_\rho \hat{A}(\cosh\beta + a_t\sinh\beta)dt \wedge d\rho \wedge d\sigma_1 \wedge d\sigma_2,$$
$$H_3 = \sinh\beta\partial_\rho \hat{A} d\rho \wedge d\sigma_1 \wedge d\sigma_2,$$
$$F_2 = \partial_\rho(a_t)dt \wedge d\rho, \quad a_t = \frac{C}{2B} = \frac{\sinh\beta\cosh\beta(1 - H)}{H\cosh^2\beta - \sinh^2\beta}, \tag{C.6}$$

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we see that we have generated a NS magnetic field. Finally, we T-dualize back in the $\vec{x}_{p-1}$ directions, to get

$$ds^2_{II,st} = g_{tt}dt^2 + \frac{d\vec{x}^2_{p-2}}{H^{1/3}B^{1/2}} + B^{1/2}\Big(H^{-2/3}d\vec{\sigma}_2^2 + H^{1/3}[d\rho^2 + \rho^2 d\Omega^2_{8-p}]\Big),$$

$$F_{p+2} = \partial_\rho \hat{A}(\cosh\beta + a_t\sinh\beta) \wedge d\rho \wedge d\sigma_1 \wedge d\sigma_2 \wedge dx_1 \wedge .... \wedge dx_{p-2},$$

$$H_3 = \sinh\beta\partial_\rho\hat{A}d\rho \wedge d\sigma_1 \wedge d\sigma_2,$$

$$F_p = \partial_\rho(a_t)dt \wedge d\rho \wedge dx_1 \wedge .... \wedge dx_{p-2},$$

$$e^{2\phi[final]} = B^{\frac{5-p}{2}}H^{\frac{2-p}{3}}. \tag{C.7}$$

This completes our derivation of eq.(2.24).

D Appendix: Another solution generating algorithm

We studied two different 'solution generating techniques' and applied them to different backgrounds. All these 'algorithms' were starting with a background solution to the Type II (A or B) equations of motion, applying a number of T-dualities that would transform the background into a solution for Type IIA supergravity. Then we lifted this to eleven dimensions, where a boost was applied (inducing a one parameter-β- family of solutions), then reducing to IIA and T-dualizing back we had our final generated background.

One may wonder what is the algorithm when, after a number of T-dualities, we end with a background solving the Type IIB supergravity equations of motion. In this case, a way of generating a one-parameter family of solutions may be S-dualizing. Indeed, given a, b, c, d real numbers satisfying $ad - bc = 1$, we can start with a IIB solution having axion χ, dilaton ϕ, RR and NS thre forms $F_3 = dC_2$ and $H_3 = dB_2$ and five form $F_5 = dC_4 + F_3 \wedge B_2$ and by S-duality, generate a new solutions dependent on the three independent real parameters a, b, c. Indeed, the five form is left invariant and the same happens for the Einstein frame metric, while

$$F_3[new] = bH_3 + aF_3, \quad H_3[new] = dH_3 + cF_3,$$

$$e^{\phi[new]} = \Big((c\chi + d)^2 + c^2e^{-2\phi}\Big)e^\phi, \quad \chi[new] = \frac{(a\chi + b)(c\chi + d) + ace^{-2\phi}}{\Big((c\chi + d)^2 + c^2e^{-2\phi}\Big)} \tag{D.1}$$

Let us focus our attention in the particular set of values $a = d = 0$, $cb = -1$, this is the transformation that interchanges the three forms and inverts the value of the dilaton (if the axion is initially zero, as we will assume).

One can then think about applying this transformation expecting to generate new interesting solutions. As an example, suppose that we start with solutions describing D6 branes wrapping a three-cycle inside the deformed conifold. Those solutions are dual to a (UV completed version of) N=1 SYM [29]. A particularly interesting solution is given in section 3.2 of

the paper [30]. The background consist of metric, dilaton and RR two-form field,

$$ds^2_{IIA,st} = e^f[-dt^2 + dx_1^2 + dx_2^2 + dx_3^2 + dM_6^2], \quad e^{2\phi} = e^{3f+2\phi_0}, \quad F_2 = dA_1 \qquad \text{(D.2)}$$

where all the details (the manifold M_6, the functional form of f, ϕ and the one form A_1 are given in eqs(57)-(59) of [30]). We can the perform three T-dualities in $x_{1,2,3}$ leading to

$$ds^2_{IIB,st} = e^f[-dt^2 + dM_6^2] + e^{-f} dx^2_{1,2,3}, \quad e^{2\phi} = e^{2\phi_0},$$
$$F_5 = dA_1 \wedge dx_1 \wedge dx_2 \wedge dx_3 \qquad \text{(D.3)}$$

that is we have generated D3 brane charge and the dilaton is just a constant. We now move this to Einstein frame, that leaves us-up to a constant-with the same metric, perform the S-duality metioned above and T-dualize back. This brings us to the starting point background (D.2)[10]. Something similar occurs if we start with a solution describing D5 branes wrapping a three-cycle inside a G2 holonomy manifold [31], a configuration dual to N=1 Yang-Mills Chern-Simons in 2+1 dimensions (with its respective UV completion). Notice that this operation is not the one proposed in [20], that is the reason why these authors were able to generate an interesting solution starting from [31].

In both these cases described above, we are generating charge of D3 brane, this is invariant under the S-duality, hence it is expected that the whole operation brings us back to the initial configuration. A more interesting example is to start from the configuration of D4 branes wrapping a circle with SUSY breaking boundary conditions [17] that we discussed before, see Section 2.1.1.

In this case we will apply three T-dualities to a IIA configuration. This will generate charge of D1 brane that the S-duality will interchange with F1 charge, after T-dualizing back, we will generate a new background (the $SO(1,3)$ isometry will be spoiled, so we may want to start with the high Temperature dual, hence having a non-extremal factor $h(\rho)$ in front of dt^2 and the function $f(\rho) = 1$ in front of dx_4^2, but let us keep things general). Let us see some details. After the T-dualities, the configuration reads

$$ds^2_{IIB,st} = H^{-1/2}(-hdt^2 + f dx_4^2) + H^{1/2}(\frac{d\rho^2}{h} + \rho^2 d\Omega_4^2 + dx^2_{1,2,3}),$$
$$e^{2\phi} = g_s^2 H, \quad F_3 = \partial_\rho A dt \wedge dx_4 \wedge d\rho \qquad \text{(D.4)}$$

now, we need to move this to Einstein frame, multiplying the metric by $e^{-\frac{\phi}{2}}$, perform the S-duality that will generate H_3 and a dilaton $e^{-2\phi} = g_s^2 H$. Then, T-dualize back in $x_{1,2,3}$. The final configurations is,

$$ds^2_{IIA,st} = \frac{c^{1/4}}{g_s H}(-hdt^2 + f dx_4^2) + \frac{d\rho^2}{h} + \rho^2 d\Omega_4^2 + dx^2_{1,2,3},$$
$$H_3 = c\partial_\rho A dt \wedge dx_4 \wedge d\rho, \quad c^4 e^{-2\phi} = g_s^2 H. \qquad \text{(D.5)}$$

---

[10]One may perform an S-duality with parameters $a = 0, bc = -1$ and d being free. This will generate a background in IIB with constant axion-dilaton. The final IIA configuration is the same as the initial one.

As a final remark; had we chosen to T-dualize only in the x_4 direction (in the supergravity approximation, this can be done in the high Temperature phase only), we would have generated D3 branes, hence after the S-duality and T-duality, we would be back to the initial configuration.

E Appendix: U-duality for the wrapped D5 branes Black Hole

We will describe in detail the action of the solution generating technique proposed in [6], when applied to the background of eq.(3.30). These techniques have been applied in a similar context in [39]. Let us consider things in the string frame.

$$
ds_s^2 = e^\phi \left[-h(\rho)dt^2 + dx_1^2 + dx_2^2 + dx_3^2 \right] + ds_{6,s}^2
$$
$$
ds_6^2 = e^\phi \left[\frac{e^{2k}}{s(\rho)}d\rho^2 + \frac{e^{2k}}{4}(\tilde\omega_3 + \cos\theta d\varphi)^2 + e^{2q}(d\theta^2 + \sin^2\theta d\varphi^2) + \frac{e^{2g}}{4}(d\tilde\theta^2 + \sin^2\tilde\theta d\tilde\varphi^2) \right],
$$
$$
F_{(3)} = \frac{N_c}{4}\left[-\tilde\omega_1 \wedge \tilde\omega_2 + \sin\theta d\theta \wedge d\varphi \right] \wedge (\tilde\omega_3 + \cos\theta d\varphi), \tag{E.1}
$$

where $h(\rho), s(\rho)$ are the non-extremality functions and we will choose $h(\rho) = s(\rho)$ as we did in section 3. We can proceed to rotate it. We will follow the procedure explained in [6]. So, let us start by writing the effect of the first T-duality in the x_1 direction (all the expressions below are in string frame)

$$
ds_{IIA}^2 = e^f \left[-hdt^2 + dx_2^2 + dx_3^2 \right] + e^{-f}dx_1^2 + ds_6^2,
$$
$$
e^{2\phi_A} = e^{2\phi - f}, \quad F_4 = F_3 \wedge dx_1 \tag{E.2}
$$

The function $f = \phi$ will be kept to avoid confusion with the transformed dilatons. Now, we perform the T-duality in x_2

$$
ds_{IIA}^B = e^f \left[-hdt^2 + dx_3^2 \right] + e^{-f}(dx_1^2 + dx_2^2) + ds_6^2,
$$
$$
e^{2\phi_B} = e^{2\phi - 2f}, \quad F_5 = F_3 \wedge dx_1 \wedge dx_2(1 + *_{10}) \tag{E.3}
$$

T-dualizing in x_3, we get

$$
ds_{IIA}^2 = e^f \left[-hdt^2 \right] + e^{-f}(dx_1^2 + dx_2^2 + dx_3^2) + ds_6^2,
$$
$$
e^{2\phi_A} = e^{2\phi - 3f},
$$
$$
F_6 = F_3 \wedge dx_1 \wedge dx_2 \wedge dx_3 \rightarrow F_4 = e^{2f}\sqrt{h} *_6 F_3 \wedge dt \tag{E.4}
$$

Notice that

$$*_6 F_3 = \frac{N_c}{\sqrt{h}} \left[-2e^{2q-2g} \sin\theta d\theta \wedge d\varphi + \frac{e^{2g-2q}}{8} \sin\tilde\theta d\tilde\theta \wedge d\tilde\varphi \right] \wedge d\rho. \qquad (E.5)$$

The factor of $\sqrt{h}$ in the F_4 of eq.(E.4) is present to cancel the factor of $\sqrt{h}$ in the denominator of eq.(E.5). Now, we lift this to M-theory;

$$ds_{11}^2 = e^{4/3\phi-2f} dx_{11}^2 + e^{f-2/3\phi} \left[-he^f dt^2 + e^{-f}(dx_1^2 + dx_2^2 + dx_3^2) + ds_6^2 \right],$$
$$G_4 = \sqrt{h}e^{2f} *_6 F_3 \wedge dt. \qquad (E.6)$$

We boost in the $t - x_{11}$ directions according to,

$$dt \to \cosh\beta dt - \sinh\beta dx_{11}, \quad dx_{11} \to -\sinh\beta dt + \cosh\beta dx_{11} \qquad (E.7)$$

and now we rewrite this boosted metric as,

$$ds_{11}^2 = e^{f-2/3\phi} \left[e^{-f}(dx_1^2 + dx_2^2 + dx_3^2) + ds_6^2 \right] + Adt^2 + Bdx_{11}^2 + Cdtdx_{11},$$
$$G_4 = \sqrt{h}e^{2f} *_6 F_3 \left[\cosh\beta dt - \sinh\beta dx_{11} \right] \qquad (E.8)$$

where,

$$A = e^{2f-2/3\phi}[\sinh^2\beta e^{2\phi-4f} - h\cosh^2\beta], \quad B = e^{2f-2/3\phi}[\cosh^2\beta e^{2\phi-4f} - h\sinh^2\beta],$$
$$C = -2\cosh\beta\sinh\beta e^{2f-2/3\phi}[e^{2\phi-4f} - h]. \qquad (E.9)$$

Now, we will reduce this to IIA, before doing so and in order to reduce to IIA, it is useful to rewrite eq.(E.8) as,

$$ds_{11}^2 = B^{-1/2} \left[g_{tt} dt^2 + B^{1/2} e^{f-2/3\phi}(e^{-f}(dx_1^2 + dx_2^2 + dx_3^2) + ds_6^2) \right] + B(dx_{11} + a_t dt)^2,$$
$$G_4 = \sqrt{h}e^{2f} *_6 F_3 \left[(\cosh\beta + a_t\sinh\beta)dt - \sinh\beta(dx_{11} + a_t dt) \right] \qquad (E.10)$$

where we have defined

$$a_t = \frac{C}{2B}, \quad g_{tt} = \frac{4AB - C^2}{4\sqrt{B}}, \quad e^{4/3\phi_A} = B. \qquad (E.11)$$

Now, we reduce to IIA, obtaining in string frame,

$$ds_{IIA}^2 = g_{tt} dt^2 + \sqrt{B}e^{-2/3\phi}(dx_1^2 + dx_2^2 + dx_3^2) + \sqrt{B}e^{f-2/3\phi} ds_6^2,$$
$$e^{2\phi_A} = B^{3/2},$$
$$F_4 = \sqrt{h}e^{2f} *_6 F_3 \wedge \left[(\cosh\beta + a_t\sinh\beta)dt \right],$$
$$H_3 = -\sinh\beta\sqrt{h}e^{2f} *_6 F_3,$$
$$F_2 = a_t' d\rho \wedge dt \qquad (E.12)$$

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Now, we proceed to do the T-dualities back; T-dualizing in the x_3 direction we have

$$ds^2_{IIB} = g_{tt}dt^2 + \sqrt{B}e^{-2/3\phi}(dx_1^2 + dx_2^2) + \frac{e^{2/3\phi}}{\sqrt{B}}dx_3^2 + \sqrt{B}e^{f-2/3\phi}ds_6^2,$$

$$e^{2\phi_B} = Be^{2/3\phi},$$

$$F_5 = \sqrt{h}e^{2f} *_6 F_3 \wedge \left[(\cosh\beta + a_t \sinh\beta)dt \wedge dx_3\right](1 + *_{10}),$$

$$H_3 = -\sinh\beta\sqrt{h}e^{2f} *_6 F_3,$$

$$F_3 = a'_t d\rho \wedge dt \wedge dx_3 \tag{E.13}$$

now, we T-dualize in x_2

$$ds^2_{IIA} = g_{tt}dt^2 + \sqrt{B}e^{-2/3\phi}(dx_1^2) + \frac{e^{2/3\phi}}{\sqrt{B}}(dx_3^2 + dx_2^2) + \sqrt{B}e^{f-2/3\phi}ds_6^2,$$

$$e^{2\phi_A} = \sqrt{B}e^{4/3\phi},$$

$$F_6 = \sqrt{h}e^{2f} *_6 F_3 \wedge \left[(\cosh\beta + a_t \sinh\beta)dt\right] \wedge dx_3 \wedge dx_2,$$

$$H_3 = -\sinh\beta\sqrt{h}e^{2f} *_6 F_3,$$

$$F_4 = a'_t d\rho \wedge dt \wedge dx_3 \wedge dx_2 \tag{E.14}$$

finally, we T-dualize in x_1

$$ds^2_{IIB} = g_{tt}dt^2 + \frac{e^{2/3\phi}}{\sqrt{B}}(dx_3^2 + dx_1^2 + dx_2^2) + \sqrt{B}e^{f-2/3\phi}ds_6^2,$$

$$e^{2\phi_B} = e^{2\phi},$$

$$F_7 = \sqrt{h}e^{2f} *_6 F_3 \wedge \left[(\cosh\beta + a_t \sinh\beta)dt\right] \wedge dx_3 \wedge dx_2 \wedge dx_1 \qquad F_3 = *_{10}F_7,$$

$$H_3 = -\sinh\beta\sqrt{h}e^{2f} *_6 F_3,$$

$$F_5 = a'_t d\rho \wedge dt \wedge dx_3 \wedge dx_2 \wedge dx_1(1 + *_{10}). \tag{E.15}$$

After using $f = \phi$ and the definitions for A, B, C, a_t this encodes the result of eq.(3.35).

F   Appendix: The equations of motion

In this appendix we will quote the equations of motion that we are numerically solving. Our goal is to find a black hole of the metrics described in the main text. In [23] the authors studied a very general Ansatz for non-extremal deformations of $NS5$ branes wrapped on S^2. Their Ansatz can be adapted to our case. It reads –in Einstein frame,

$$ds^2 = -Y_1 dt^2 + Y_2 dx^n dx^n + Y_3 d\rho^2 + Y_4 (e_1^2 + e_2^2) + Y_5 \left((w^1)^2 + (w^2)^2\right) + Y_6 (w^3 + A)^2 \tag{F.1}$$

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Inserting this ansatz in the supergravity action we get,

$$L = \sum_{i,j} G_{ij}(Y)Y_i'Y_j' - U(Y) = T - U \tag{F.2}$$

Using a parametrization to make G_{ij} diagonal and choosing the appropiate gauge to make contact with our ansatz we have,

$$T = e^{2(g+q-4x+\Phi)}\frac{1}{8}\left(\frac{1}{2}\left(g'^2 + q'^2 - 2\Phi'^2\right) + 2(g'q' - k'x') + (g' + q' + \Phi')(k' - 4x' + 2\Phi')\right),$$

$$U = \frac{1}{256}e^{-2(g+q-\Phi)}\left(-16e^{2(g+q+k)}(e^{2g} + 4e^{2q}) + (e^{4g} + 16e^{4q})(1 + e^{4k})\right). \tag{F.3}$$

and,

$$Y_1 = e^{\Phi/2}e^{-8x}, \qquad Y_2 = e^{\Phi/2}, \qquad Y_3 = e^{\Phi/2}e^{8x}e^{2k},$$
$$Y_4 = e^{\Phi/2}e^{2g}, \qquad Y_5 = e^{\Phi/2}e^{2q}. \qquad Y_6 = \Phi \tag{F.4}$$

¿From (F.2) we get the second order equations to solve,

$$2e^{-4g+8x} + \frac{1}{8}e^{-4q+8x} - 2\Phi'(g' + q' - 4x' + \Phi') - \Phi'' = 0$$

$$-2x'(g' + q' - 4x' + \Phi') - x'' = 0$$

$$-e^{4k+8x}(2e^{-4g} + \frac{1}{8}e^{-4q}) + e^{8x}(2e^{-4g} + \frac{1}{8}e^{-4q}) + 2k'(g' + q' - 4x' + \Phi') + k'' = 0 \tag{F.5}$$

$$2e^{8x}(e^{4k-4g} - 2e^{2(k-g)} + 1) + g'(g' + q' - 4x' + \Phi') + g'' = 0$$

$$e^{8x}(-\frac{1}{8}e^{4k-4q} + e^{2k-2q} - \frac{1}{8}e^{-4q}) - 2q'(g' + q' - 4x' + \Phi') - q'' = 0$$

and a first order constraint which is a consequence of reparametrization invariance,

$$\frac{e^{8x}}{2}\left(\frac{1}{16}(e^{-4q} + 16e^{-4g})(1 + e^{4k}) - e^{2k}(e^{-2q} + 4e^{-2g})\right) + \frac{1}{2}(g'^2 + q'^2 - 2\Phi'^2) +$$
$$2(g'q' - k'x') + (g' + q' + \Phi')(k' - 4x' + 2\Phi') = 0 \tag{F.6}$$

The second equation in (F.5) can be integrated to yield a first order equation and (F.5) becomes

$$2e^{-4g+8x} + \frac{1}{8}e^{-4q+8x} - 2\Phi'(g' + q' - 4x' + \Phi') - \Phi'' = 0$$

$$x' - \alpha e^{-2(g+q-4x+\Phi)} = 0$$

$$-e^{4k+8x}(2e^{-4g} + \frac{1}{8}e^{-4q}) + e^{8x}(2e^{-4g} + \frac{1}{8}e^{-4q}) + 2k'(g' + q' - 4x' + \Phi') + k'' = 0 \tag{F.7}$$

$$e^{8x}(e^{4k-4g} - 2e^{2(k-g)} + 1) + g'(g' + q' - 4x' + \Phi') + g'' = 0$$

$$e^{8x}(-\frac{1}{8}e^{4k-4q} + e^{2k-2q} - \frac{1}{8}e^{-4q}) - 2q'(g' + q' - 4x' + \Phi') - q'' = 0$$

where α is a non-extremality parameter.

One can verify that the transformation

$$r \to e^{2d}r \qquad \Phi \to \Phi + d \qquad x \to x + \frac{d}{2} \qquad g \to g \qquad q \to q \qquad k \to k \qquad \text{(F.8)}$$

where d is a constant, leaves (F.5) and (F.6) invariant and thus, is a symmetry of the equations of motion and constraint. Another, obvious, symmetry of (F.5) and (F.6) is

$$\Phi \to \Phi + C \qquad \alpha \to \alpha e^{2C} \qquad \text{(F.9)}$$

with all the other functions unchanged and C a constant. Finally, there is also a translational symmetry

$$r \to r + r_0 \qquad \text{(F.10)}$$

that leaves all equations unchanged. Note that thermodynamic quantities should be invariant under all these symmetries.

References

[1] J. M. Maldacena, Adv. Theor. Math. Phys. **2**, 231 (1998) [Int. J. Theor. Phys. **38**, 1113 (1999)] [arXiv:hep-th/9711200].

[2] S. S. Gubser, I. R. Klebanov and A. M. Polyakov, Phys. Lett. B **428**, 105 (1998) [arXiv:hep-th/9802109]. E. Witten, Adv. Theor. Math. Phys. **2**, 253 (1998) [arXiv:hep-th/9802150]. N. Itzhaki, J. M. Maldacena, J. Sonnenschein and S. Yankielowicz, Phys. Rev. D **58**, 046004 (1998) [arXiv:hep-th/9802042].

[3] O. Lunin and J. M. Maldacena, JHEP **0505**, 033 (2005) [arXiv:hep-th/0502086]. J. Maldacena, D. Martelli and Y. Tachikawa, JHEP **0810**, 072 (2008) [arXiv:0807.1100 [hep-th]]. C. P. Herzog, M. Rangamani and S. F. Ross, JHEP **0811**, 080 (2008) [arXiv:0807.1099 [hep-th]]. C. P. Herzog, M. Rangamani and S. F. Ross, JHEP **0811**, 080 (2008) [arXiv:0807.1099 [hep-th]].

[4] V. Belinski and E. Verdaguer, *Cambridge, UK: Univ. Pr. (2001) 258 p*A. A. Pomeransky, Phys. Rev. D **73**, 044004 (2006) [arXiv:hep-th/0507250]. T. Mishima and H. Iguchi, Phys. Rev. D **73**, 044030 (2006) [arXiv:hep-th/0504018].

[5] H. Elvang and P. Figueras, JHEP **0705**, 050 (2007) [arXiv:hep-th/0701035].

[6] J. Maldacena and D. Martelli, JHEP **1001**, 104 (2010) [arXiv:0906.0591 [hep-th]].

[7] A. Butti, M. Grana, R. Minasian, M. Petrini and A. Zaffaroni, JHEP **0503**, 069 (2005) [arXiv:hep-th/0412187].

[8] I. R. Klebanov and M. J. Strassler, JHEP **0008**, 052 (2000) [arXiv:hep-th/0007191].

[9] R. Casero, C. Nunez and A. Paredes, Phys. Rev. D **73**, 086005 (2006) [arXiv:hep-th/0602027]. R. Casero, C. Nunez and A. Paredes, Phys. Rev. D **77**, 046003 (2008) [arXiv:0709.3421 [hep-th]].

[10] C. Hoyos-Badajoz, C. Nunez and I. Papadimitriou, Phys. Rev. D **78**, 086005 (2008) [arXiv:0807.3039 [hep-th]].

[11] A. H. Chamseddine and M. S. Volkov, Phys. Rev. Lett. **79**, 3343 (1997) [arXiv:hep-th/9707176]. J. M. Maldacena and C. Nunez, Phys. Rev. Lett. **86**, 588 (2001) [arXiv:hep-th/0008001].

[12] R. Minasian, M. Petrini and A. Zaffaroni, JHEP **1004**, 080 (2010) [arXiv:0907.5147 [hep-th]].

[13] J. Gaillard, D. Martelli, C. Nunez and I. Papadimitriou, Nucl. Phys. B **843**, 1 (2011) [arXiv:1004.4638 [hep-th]].

[14] N. Halmagyi, arXiv:1003.2121 [hep-th].

[15] A. Dymarsky, I. R. Klebanov and N. Seiberg, JHEP **0601**, 155 (2006) [arXiv:hep-th/0511254].

[16] R. P. Andrews and N. Dorey, Nucl. Phys. B **751**, 304 (2006) [arXiv:hep-th/0601098]. R. P. Andrews and N. Dorey, Phys. Lett. B **631**, 74 (2005) [arXiv:hep-th/0505107].

[17] E. Witten, Adv. Theor. Math. Phys. **2**, 505 (1998) [arXiv:hep-th/9803131].

[18] T. Sakai and S. Sugimoto, Prog. Theor. Phys. **113**, 843 (2005) [arXiv:hep-th/0412141].

[19] O. Aharony, J. Sonnenschein and S. Yankielowicz, Annals Phys. **322**, 1420 (2007) [arXiv:hep-th/0604161].

[20] J. Gaillard and D. Martelli, arXiv:1008.0640 [hep-th].

[21] C. Nunez, A. Paredes and A. V. Ramallo, Adv. High Energy Phys. **2010**, 196714 (2010) [arXiv:1002.1088 [hep-th]].

[22] C. Nunez, I. Papadimitriou and M. Piai, Int. J. Mod. Phys. A **25**, 2837 (2010) [arXiv:0812.3655 [hep-th]].

[23] S. S. Gubser, A. A. Tseytlin, and M. S. Volkov, *JHEP* **0109:017,2001** (JHEP 0109:017,2001).

[24] E. Caceres, R. Flauger, M. Ihl, and T. Wrase, *JHEP* **0803:020,2008** (Nov., 2007).

[25] E. Caceres, R. Flauger, and T. Wrase, arXiv:0908.4483.

[26] G. T. Horowitz, A. Strominger, Nucl. Phys. **B360**, 197-209 (1991).

[27] M. J. Duff, J. X. Lu, Nucl. Phys. **B416**, 301-334 (1994). [hep-th/9306052].

[28] M. J. Duff, R. R. Khuri, J. X. Lu, Phys. Rept. **259**, 213-326 (1995). [hep-th/9412184].

[29] M. Atiyah, J. M. Maldacena and C. Vafa, J. Math. Phys. **42** (2001) 3209 [arXiv:hep-th/0011256]. C. Vafa, J. Math. Phys. **42**, 2798 (2001) [arXiv:hep-th/0008142]. J. D. Edelstein and C. Nunez, JHEP **0104**, 028 (2001) [arXiv:hep-th/0103167].

[30] A. Brandhuber, Nucl. Phys. B **629**, 393 (2002) [arXiv:hep-th/0112113].

[31] J. M. Maldacena and H. S. Nastase, JHEP **0109**, 024 (2001) [arXiv:hep-th/0105049].

[32] C. G. . Callan, J. A. Harvey and A. Strominger, Nucl. Phys. B **359** (1991) 611.

[33] A. Buchel, Nucl. Phys. B **600** (2001) 219 [arXiv:hep-th/0011146]. A. Buchel, C. P. Herzog, I. R. Klebanov, L. A. Pando Zayas and A. A. Tseytlin, JHEP **0104** (2001) 033 [arXiv:hep-th/0102105]. S. S. Gubser, C. P. Herzog, I. R. Klebanov and A. A. Tseytlin, JHEP **0105** (2001) 028 [arXiv:hep-th/0102172]. L. A. Pando Zayas and C. A. Terrero-Escalante, JHEP **0609** (2006) 051 [arXiv:hep-th/0605170]. O. Aharony, A. Buchel and P. Kerner, Phys. Rev. D **76** (2007) 086005 [arXiv:0706.1768 [hep-th]]. M. Mahato, L. A. Pando Zayas and C. A. Terrero-Escalante, JHEP **0709** (2007) 083 [arXiv:0707.2737 [hep-th]]. A. Buchel, arXiv:1012.2404 [hep-th].

[34] G. T. Horowitz and D. L. Welch, Phys. Rev. D **49**, 590 (1994) [arXiv:hep-th/9308077].

[35] M. Schvellinger, JHEP **0409**, 057 (2004) [arXiv:hep-th/0407152]. P. McGuirk, G. Shiu and Y. Sumitomo, Nucl. Phys. B **842**, 383 (2010) [arXiv:0910.4581 [hep-th]].

[36] I. Bena, M. Grana and N. Halmagyi, JHEP **1009**, 087 (2010) [arXiv:0912.3519 [hep-th]]. I. Bena, G. Giecold and N. Halmagyi, arXiv:1011.2195 [hep-th].

[37] J. Gaillard and J. Schmude, JHEP **1002**, 032 (2010) [arXiv:0908.0305 [hep-th]].

[38] F. Bigazzi, A. L. Cotrone, J. Mas, A. Paredes, A. V. Ramallo and J. Tarrio, JHEP **0911**, 117 (2009) [arXiv:0909.2865 [hep-th]]. F. Bigazzi, A. L. Cotrone and J. Tarrio, JHEP **1002**, 083 (2010) [arXiv:0912.3256 [hep-th]]. F. Bigazzi and A. L. Cotrone, JHEP **1008**, 128 (2010) [arXiv:1006.4634 [hep-ph]]. M. Mia, K. Dasgupta, C. Gale and S. Jeon, Nucl. Phys. B **839** (2010) 187 [arXiv:0902.1540 [hep-th]]. F. Bigazzi, A. L. Cotrone, J. Mas, D. Mayerson and J. Tarrio, arXiv:1101.3560 [hep-th].

[39] F. Chen, K. Dasgupta, P. Franche, S. Katz and R. Tatar, arXiv:1007.5316 [hep-th].

Heating up the Baryonic Branch with U-duality: a unified picture of conifold black holes

Elena Cáceres[*1], Carlos Núñez[†2] and Leopoldo A. Pando Zayas [**3]

[*] *Facultad de Ciencias*
Universidad de Colima
Bernal Diaz del Castillo 340, Colima, México.
and
Theory Group, Department of Physics,
University of Texas at Austin, Austin, TX 78727, USA.

[†] *Department of Physics*
University of Swansea, Singleton Park
Swansea SA2 8PP
United Kingdom.

[**] *Michigan Center for Theoretical Physics*
Randall Laboratory of Physics, the University of Michigan
Ann Arbor, MI 48109-1040. USA

Abstract

We study different aspects of a U-duality recently presented by Maldacena and Martelli and apply it to non-extremal backgrounds. In particular, starting from new non-extremal wrapped D5 branes we generate new non-extremal generalizations of the Baryonic Branch of the Klebanov-Strassler solution. We also elaborate on different conceptual aspects of these U-dualities, like its action on (extremal and non-extremal) Dp branes, dual models for Yang-Mills-like theories, generic asymptotics and decoupling limit of the generated solutions.

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[1]elenac@zippy.ph.utexas.edu
[2]c.nunez@swansea.ac.uk
[3]lpandoz@umich.edu

Contents

1 Introduction

The Maldacena conjecture [1] and some of its refinements [2] are the guiding principle behind much of the progress in the interface String Theory-Quantum Field theory in the last twelve years. The influence of this approach extends to toy models with different number of supersymmetries (SUSY's), systems at finite temperature and/or finite density and lower dimensional systems. The applications to physically relevant systems do not seem to be exhausted and on the contrary, increase with time. In this sense, finding new (trustable) solutions to the equations of motion of String Theory (even in the point particle and classical approximation) has become an industry with various applications in Physics and Mathematics. Solution generating techniques have certainly played a role. Some examples worth mentioning are the combination of T-dualities and shifts of coordinates that generated solutions dual to minimally SUSY superconformal field theories (beta deformations) or duals to non-relativistic field theories [3][1].

In this paper we will focus our attention on a particular solution generating technique that was presented in [6]. The procedure suggested by these authors starts by taking a solution in type IIB string theory proposed to be dual to (a suitably UV-completed version of) $\mathcal{N} = 1$ Super Yang-Mills in four dimensions. The solution has the topology $R^{1,3} \times M_6$ where M_6 preserves four supercharges. The algorithm to generate the new solutions goes as follows: first apply a set of three T-dualities in the R^3 directions, which leaves us with a IIA configuration; lift this configuration to M-theory and boost–with rapidity β–in the eleventh direction; then reduce to IIA and T-dualize back in R^3. This generates a solution that roughly speaking is the dual description to the baryonic branch [7] of the Klebanov-Strassler field theory [8]. For all the technical details of this procedure the reader can refer to [6] or to our Appendix E.

The 'seed solution' (as we will refer to the initial solution on which the generating algorithm is applied) was discussed in the papers [9] (Section 8) and [10] (Section 4.3). In a particular limit, this becomes the exact solution discussed in [11].

We will also refer to the solution generating algorithm, that is a U-duality, as 'rotation' in a sense explained below and in previous papers. In the following we will emphasize some aspects of the generating technique presented in [6] that we find particularly interesting and have not been explicitly discussed in the existing bibliography:

- In the case of [6], the rotation generates D3 brane charge. This is the reason why the $SO(1,3)$ isometry of the background is untouched, in spite of doing *different* operations in time (boost) and in the R^3 directions (T-dualities). As a by-product of the generation of D3 branes the dilaton is invariant under this set of operations.

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[1]In other areas of Physics, solution generating techniques play a major role. For example, many of the interesting solutions in higher dimensional gravity were constructed using solution generating algorithms. See for example [4] for original papers and [5] for a nice summary with an important application.

- It can be seen that this U-duality (sequence of T-dualities, lifts and boosts) described above is equivalent to a *particular case* of 'rotation' in the $SU(3)$ structure of the manifold M_6 which is characterized by a complex 3-form Ω_3 and a 2-form J_2 - see the papers [12] and [13]. This is another way of understanding the presence of the $SO(1,3)$ isometry: from this perspective, all the rotation 'occurs' in the internal M_6.

- The rotation generates a solution that contains two free parameters, the value of the dilaton at infinity (already present in the seed solution) and the boost parameter β. In order for the final background to be dual to the Klebanov-Strassler QFT *decoupled* from gravity, we must take the limit $\beta \to \infty$ so that the generated warp factor vanishes asymptotically. In this sense, the seed solution describes a field theory coupled to gravity. Only in a particular limit-see the discussion in [6], [13], we approach the near brane solution of [11].

- This rotation or U-duality, is a very curious operation from the viewpoint of the dual field theory. Indeed, it generates (aside from the bifundamental fields) new global symmetries, like the baryonic $U(1)$.

- In the same vein: the connection between the two field theories from a geometric viewpoint (Field theory at strong coupling!) is just the mentioned U-duality starting from the $D5$-brane side. From a (weakly interacting) field theoretic viewpoint the operation is very subtle [6]. Indeed, it involves a second order fluctuation expansion (mass spectrum) in the classical and weakly coupled F and D term equations on the quiver side [15], that is matched with a classical and weakly coupled expansion of the twisted compactification of the theory in the wrapped D5 brane mass spectrum [16].

- After the first set of T-dualities described above, we have a IIA background with D2 brane charges. When lifted to M-theory we have M2 branes, that after boosted generate M2 charge and an M-wave. When reduced, this generates a D0, D2 and NS H_3 field. The generation of the H_3 in the presence of RR three-form implies the need to generate F_5. See Appendix E for technical details.

- We emphasized that the U-duality (chain of T-dualities, lifts and boost) described above is a *particular case* of the rotation of J_2, Ω_3 discussed in [12], [13], [14]. On the other hand, the rotation of J_2, Ω_3 relies on the background being supersymmetric, while the U-duality can be also applied to Non-SUSY backgrounds. This will play an important role in the rest of this paper.

1.1   General Idea of this Paper

In this paper we will try to gain a different perspective on this solution generating technique. We will not focus on the rotation of J, Ω but rely on the U-duality description (in spite of the

latter being a particular case of the former). Our interests will be two-fold. On one hand, we will try to get a better handle on the generating algorithm by changing it (suggesting other related algorithms) and applying it to various cases. On the other hand, we will apply it to backgrounds that do not preserve SUSY. We will study the effect of the rotation on black hole solutions, generating *new* non-extremal solutions with horizons and other curious features that will be discussed. We will also start the study of the properties of those newly generated solutions.

This paper is organized as follows: In sections 2 and 2.1 as a warm-up exercise we discuss a chain of dualities and boost acting on non-extremal Dp branes. In section 2.1.1 we will apply this to rotate a background dual to a version of Yang-Mills. Then, in section 2.2 we will study a variation of this U-duality that will clarify various aspects of the coming material. In Section 3 we will U-dualize a new solution describing the non-extremal deformation of a stack of N_c D5 branes wrapping a two-cycle inside the resolved conifold. We will elaborate upon various aspects of this particular rotation in section 3.5. Finally in Section 4 we comment on decoupling aspects of the rotation. Various appendices complement our presentation. They have been written with plenty of detail hoping that colleagues working on these topics will find them useful. We close with a summary, conclusions and a list of possible future projects.

2 A warm-up example: Rotation of Dp-branes

In this section, we will start with a simple example of the 'rotation' procedure. That is, a sequence of T-dualities, bringing the background to a type IIA solution of the supergravity equations of motion, followed by a lift to eleven dimensions, where we will boost the configuration. We will then reduce to IIA and T-dualize back, to what we will call the 'rotated background'. As a toy example, in this section we will rotate flat Dp branes; first in p directions and then in $p - q$ directions. We will do this in detail to appreciate the differences this introduces in the generated solution. In Appendix D, we will propose and analyze another possible U-duality to generate new solutions.

2.1 "Rotation" in p directions

Consider backgrounds of IIA/IIB of the form,

$$ds^2 = H(\rho)^{-1/2}\Big[-h(\rho)dt^2 + d\vec{x}_p^2\Big] + H(\rho)^{1/2}\Big[\frac{d\rho^2}{h(\rho)} + \rho^2 d\Omega_{8-p}\Big],$$

$$e^{2\phi[initial]} = e^{2\phi(\infty)}H^{\frac{3-p}{2}},$$

$$*F_{p+2} = \mathcal{Q}\,Vol_{\,\Omega_{8-p}},$$

$$F_{p+2} = -\partial_\rho A(\rho)dt \wedge dx_1 \wedge .... \wedge dx_p \wedge d\rho \tag{2.1}$$

with,

$$H(\rho) = 1 + \left(\frac{L_p}{\rho}\right)^{7-p}, \quad h(\rho) = 1 - \left(\frac{R_T}{\rho}\right)^{7-p}, \quad A(\rho) = \frac{\alpha}{g_s H(\rho)}, \tag{2.2}$$

where $\alpha = \frac{\tilde{Q}}{L_p^{7-p}}$ and $\mathcal{Q} = (7-p)\tilde{Q}$ is related to the charge of the solution (see Appendix B for details). If we choose $R_T = 0$, the configuration above is typically singular (except for $p = 3$) and preserves 16 SUSYs. Recall that for even (odd) values of p, we are dealing with a solution of Type IIA(B) supergravity.

Now, we will 'rotate' this background, by first T-dualizing in the p directions, this will bring us to a IIA solution, we will lift the solution to eleven-dimensional supergravity and perform a boost of rapidity β in the eleventh-direction. We will then reduce to IIA and T-dualize back in the $\vec{x}_p$ coordinates to obtain what we call the 'rotated background'. Our final rotated background is given by[2],

$$ds^2 = \frac{H(\rho)^{-1/2}}{R(\rho)}\left[-h(\rho)dt^2 + d\vec{x}_p^2\right] + R(\rho)H(\rho)^{1/2}\left[\frac{d\rho^2}{h(\rho)} + \rho^2 d\Omega_{8-p}\right] \tag{2.3}$$

$$e^{2\phi[final]} = e^{2\phi[initial]}R(\rho)^{3-p} = e^{2\phi(\infty)}(H(\rho)R(\rho)^2)^{\frac{3-p}{2}} \tag{2.4}$$

$$F_{p+2} = -\partial_\rho a(\rho)dt \wedge dx_1 \wedge .... \wedge dx_p \wedge d\rho \tag{2.5}$$

where we have defined,

$$R^2 = (A(\rho)\sinh\beta + \cosh\beta)^2 - \frac{h(\rho)\sinh^2\beta}{g_s^2 H(\rho)^2}, \tag{2.6}$$

$$a = \frac{1}{R^2}[A(\rho)\cosh 2\beta + (A(\rho)^2 + 1)\cosh\beta\sinh\beta - \frac{h(\rho)}{2g_s^2 H(\rho)^2}\sinh(2\beta)] \tag{2.7}$$

We have explicitly checked that the rotated background is a solution of the equations of motion. Notice that eq. (2.6) can be written as

$$R^2 = \cosh^2\beta + A(\rho)\sinh 2\beta + (\frac{R_T}{L_p})^{7-p}\frac{\sinh^2\beta}{H(\rho)g_s^2} \tag{2.8}$$

which makes clear that R is strictly positive. In a similar way, eq. (2.7) can be written as

$$a = \frac{1}{HR^2}\left[\frac{1}{g_s}\alpha\cosh(2\beta) + \left(\frac{R_T}{L_p}\right)^{7-p}\frac{\sinh\beta\cosh\beta}{g_s^2} + H(\rho)\sinh\beta\cosh\beta\right]. \tag{2.9}$$

The metric (2.3) has the structure of a warped space,

$$ds^2 = \mathcal{H}(\rho)^{-1/2}\left[-h(\rho)dt^2 + d\vec{x}_p^2\right] + \mathcal{H}(\rho)^{1/2}\left[\frac{d\rho^2}{h(\rho)} + \rho^2 d\Omega_{8-p}\right], \tag{2.10}$$

---

[2]In Appendix A, we will present the intermediate steps of this calculation for the extremal case $h = 1$.

where the new warp factor is

$$\mathcal{H}(\rho) = H(\rho)R(\rho)^2 = H(\rho)\cosh^2\beta + \frac{\alpha}{g_s}\sinh 2\beta + (\frac{R_T}{L_p})^{7-p}\frac{\sinh^2\beta}{g_s^2} \qquad (2.11)$$

Note that $\mathcal{H}(\rho)$ is a harmonic function of the transverse space. Hence, after the rotation in p directions we are left with a Dp brane solution. Indeed, in terms of the new warp factor, the dilaton and gauge potential are

$$e^{2\phi[final]} = e^{2\phi(\infty)}\mathcal{H}(\rho)^{\frac{3-p}{2}}$$

$$\partial_\rho a(\rho) = \left[\frac{\alpha}{g_s} + (\frac{R_T}{L_p})^{7-p}\frac{\tanh\beta}{g_s^2}\right]\ \partial_\rho\Big(\frac{1}{\mathcal{H}(\rho)}\Big) = \tilde{\alpha}\ \partial_\rho\Big(\frac{1}{\mathcal{H}(\rho)}\Big), \qquad (2.12)$$

which together with (2.10) define a Dp brane background with a β dependent RR charge. At infinity the warp factor asymptotes to a constant,

$$\mathcal{H}(\rho) \sim \cosh^2\beta + \frac{\alpha}{g_s}\sinh 2\beta + (\frac{R_T}{L_p})^{7-p}\frac{\sinh^2\beta}{g_s^2}, \qquad (2.13)$$

and the space is asymptotically flat, as expected. However, it is interesting to note that even if we start with the Dp branes after the decoupling limit is taken, that is $H = \frac{L_p^{7-p}}{\rho^{7-p}}$, we will have in the UV that,

$$\mathcal{H}(\rho) \sim \frac{L_p^{7-p}}{\rho^{7-p}}\cosh^2\beta + \frac{\alpha}{g_s}\sinh 2\beta + (\frac{R_T}{L_p})^{7-p}\frac{\sinh^2\beta}{g_s^2} \qquad (2.14)$$

So, again, the warp factor asymptotes to a constant. In other words, this rotation is taking the configuration *out of the decoupling limit* or coupling the field theory modes to gravity.

As is well known, before the rotation, the charge of the Dp brane solution is quantized. After the rotation the charge is,

$$\frac{1}{(2\pi l_s)^{7-p}}\int_{S^{8-p}} *F_{p+2} = \tilde{\alpha}\ \rho^{8-p}\mathcal{H}'(\rho)Vol_{S^{8-p}}$$

$$= \mathrm{Q}\cosh^2\beta + R_T^{7-p}\frac{c_p}{2g_s}\sinh 2\beta \qquad (2.15)$$

where $c_p = \frac{(7-p)Vol_{S^{8-p}}}{(2\pi l_s)^{7-p}g_s}$. Note that, generically, the charge is not quantized. This is not unusual since the supergravity dualities involved in the rotation procedure are a symmetry of the supergravity equations and not of the full string theory.

We will now move to study a more interesting example from the viewpoint of gauge-strings duality. We will apply the rotation procedure to the a non-SUSY Yang-Mills-like theory, first presented in [17] and further studied in [18], [19].

2.1.1   Example: rotation of a dual to a Yang-Mills-like theory

The original background, consists of the decoupling limit of a stack of N_c D4 branes wrapping a circle with SUSY breaking boundary conditions. It was discussed with details in various publication, see for example [17], [18], [19]. We summarize it here (in string frame)[3]

$$ds^2 = H(\rho)^{-1/2}\Big[-h(\rho)dt^2 + d\vec{x}_{123}^2 + f(\rho)dx_4^2\Big] + H(\rho)^{1/2}\Big[\frac{d\rho^2}{f(\rho)h(\rho)} + \rho^2 d\Omega_4\Big],$$

$$e^{2\phi[initial]} = g_s^2 H^{-\frac{1}{2}},$$

$$F_6 = -\partial_\rho A(\rho)dt \wedge dx_1 \wedge \dots \wedge dx_4 \wedge d\rho \quad F_4 = *F_6 \tag{2.16}$$

where $A(\rho) = \frac{1}{g_s H(\rho)}$ and Vol_4 is the volume of the unit four- sphere Ω_4.. The functions (in the low/zero temperature phase) are given by

$$H = (\frac{L}{\rho})^3, \quad f = 1 - (\frac{R_{kk}}{\rho})^3, \quad h = 1, \tag{2.17}$$

where $L^3 = N_c \pi g_s l_s^3$ and the coefficient R_{kk} is a free parameter. In the high temperature phase (with the coordinate x_4 compactified) we have,

$$H = (\frac{L}{\rho})^3, \quad h = 1 - (\frac{R_T}{\rho})^3, \quad f = 1 \tag{2.18}$$

where R_T is a free parameter related to the temperature of the system,

$$T = \frac{3}{4\pi}(\frac{L^3}{R_T})^{-1/2} \tag{2.19}$$

Proceeding as described in the previous section we rotate the background by applying the U-duality already discussed: four T-dualities in the worldvolume coordinates, uplift to M-theory, boost, reduce to IIA and T dualize back. The rotated background is,

$$ds^2 = \mathcal{H}(\rho)^{-1/2}\Big[-h(\rho)dt^2 + d\vec{x}_{123}^2 + f(\rho)dx_4^2\Big] + \mathcal{H}(\rho)^{1/2}\Big[\frac{d\rho^2}{f(\rho)h(\rho)} + \rho^2 d\Omega_4\Big],$$

$$e^{2\phi[final]} = g_s^2 \mathcal{H}^{-\frac{1}{2}},$$

$$F_6 = a'(\rho)dt \wedge dx_1 \wedge \dots \wedge dx_4 \wedge d\rho \tag{2.20}$$

where the functions $\mathcal{H}(\rho), a(\rho)$ now read

$$\mathcal{H}(\rho) = H(\rho)\cosh^2\beta + \frac{1}{g_s}\sinh 2\beta + (\frac{R_T}{L})^3\frac{\sinh^2\beta}{g_s^2} \tag{2.21}$$

$$a(\rho) = \frac{1}{R(\rho)^2}\Big[A(\rho)\cosh 2\beta + (A(\rho)^2 + 1)\cosh\beta\sinh\beta - \frac{f(\rho)h(\rho)\sinh(2\beta)}{2g_s^2 H(\rho)^2}\Big] \tag{2.22}$$

---

[3] In [18] the authors performed a rescaling of the RR potential: $C_{p+1} \to \frac{\kappa_0^2 \mu_{6-p}}{\pi} C_{p+1}$ and as a result the RR charge in their paper is measured in units of 2π. We do not perform such rescaling here.

and $R(\rho)^2 = \frac{\mathcal{H}(\rho)}{H(\rho)}$. As before, one can check explicitly that the background above is a solution of the eqs. of motion, but for this we must set either $R_T = 0$ for the low/zero temperature phase or set $R_{kk} = 0$, compactifying the x_4 direction, in the high temperature phase.

As explained in the previous section, in spite of having started with a decoupled background, after the rotation we have an asymptotically flat space, indicating that the rotation has coupled the field theory modes to the gravitational ones.

2.2 "Rotation" in p-q directions

We will study now a peculiar situation, where we have Dp branes and we separate a q manifold from the $p + 1$ dimensional world-volume. The internal manifold need not be specified. We will impose on our U-duality that:

- after the initial T-dualities, we are in IIA background, so that this is upliftable to M-theory

- that the boost generates a NS H_3 field, or what is equivalent, that the IIA configuration after the initial T-dualities contains an electric F_4.

We will assume (though this need not be the case) that the internal q-manifold is a torus. In that case, the solution reads (in string frame as usual)[4],

$$ds^2 = H^{-1/2}[-dt^2 + dx_{p-q}^2 + d\sigma_q^2] + H^{1/2}[d\rho^2 + \rho^2 d\Omega_{8-p}^2],$$
$$F_{p+2} = \partial_\rho \hat{A} dt \wedge dx_1 \wedge ... \wedge dx_{p-q} \wedge d\sigma_q \wedge d\rho,$$
$$e^{2\phi[initial]} = e^{2\phi(\infty)} H^{\frac{3-p}{2}}. \tag{2.23}$$

We will proceed as follows: first we will do T dualities in the x_{p-q} directions, this will leave us in a IIA configuration, we will lift then to M-theory and boost. We reduce to IIA and T dualize back, all the details are written in Appendix C. We end up this duality-chain with a final configuration

$$ds_{II,st}^2 = g_{tt}dt^2 + \frac{d\vec{x}_{p-2}^2}{H^{1/3}B^{1/2}} + B^{1/2}\left(H^{-2/3}d\vec{\sigma}_2^2 + H^{1/3}[d\rho^2 + \rho^2 d\Omega_{8-p}^2]\right),$$
$$F_{p+2} = \partial_\rho \hat{A}(\cosh\beta + a_t \sinh\beta) \wedge d\rho \wedge d\sigma_1 \wedge d\sigma_2 \wedge dx_1 \wedge .... \wedge dx_{p-2},$$
$$H_3 = \sinh\beta \partial_\rho \hat{A} d\rho \wedge d\sigma_1 \wedge d\sigma_2,$$
$$F_p = \partial_\rho(a_t)dt \wedge d\rho \wedge dx_1 \wedge .... \wedge dx_{p-2},$$
$$e^{2\phi[final]} = B^{\frac{5-p}{2}} H^{\frac{2-p}{3}} \tag{2.24}$$

---

[4]we are dealing here with the Lorentz invariant case here. Working with a non-extremal factor is straightforward.

where we have defined (See Appendix C for a detailed derivation),

$$A = H^{-2/3}[H \sinh^2 \beta - \cosh^2 \beta], \quad B = H^{-2/3}[H \cosh^2 \beta - \sinh^2 \beta],$$

$$C = 2H^{-2/3} \sinh \beta \cosh \beta (1 - H), \quad g_{tt} = \frac{4AB - C^2}{4\sqrt{B}},$$

$$a_t = \frac{C}{2B} = \frac{\sinh \beta \cosh \beta (1 - H)}{H \cosh^2 \beta - \sinh^2 \beta}, \tag{2.25}$$

Various aspects are worth emphasizing in this final background. Notice that:

- the metric is dual to a field theory with $SO(1, p-2)$ isometry, since

$$(4AB - C^2)H^{1/3} = -4 \rightarrow g_{tt} = -g_{x_i x_i} \tag{2.26}$$

- for $p = 5$ the dilaton is invariant under the whole U-duality procedure, since

$$e^{2\phi[initial]} = H^{\frac{3-p}{2}} = e^{2\phi[final]} = B^{\frac{5-p}{2}} H^{\frac{2-p}{3}} \tag{2.27}$$

- for $p = 5$ the procedure is generating charge of D3 brane (represented by the $F_{p=5}$) where the D3 branes are extended in the $p - q$ directions. This is the reason why the dilaton does not change as the D3 branes do not couple to it. The field H_3 is also generated.

In following sections, we will study a particular case of this U-duality, for a situation in which a set of D5 branes wrap a curved two-manifold (but we also have a black hole in the metric, breaking the $SO(1, 3)$ isometry), the results are qualitatively the same. One should emphasize that there is yet another way of generating NS three-form fields, that is basically starting from NS five branes in IIA wrapping a three-cycle, see [20] for details.

Also, notice that we have imposed that after the first set of T-dualities, the background is solution of IIA Supergravity (the conditions for this to happen are discussed in Appendix C). Were this not the case, we present in Appendix D another possible solution generating technique with some applications.

Now, we will move to study an interesting application of the U-dualities discussed above.

3 Black Holes in $\mathcal{N} = 1$ SUSY Solutions

As anticipated, in this section we will apply the chain of dualities, lift and boost proposed in [6], to generate a new solution starting from a non-extremal solution in Type IIB. The interest of the original ('seed') solution is that it was argued to be dual to a field theory with minimal SUSY in four dimensions. The field theory was studied (at weak coupling) in [16] and is basically $\mathcal{N} = 1$ Super-Yang-Mills plus a sets of (massive) KK vector and chiral superfields

that UV complete the dynamics. This system was well studied and various string duals are known (particular solutions describing the field theory at strong coupling, with VEV's and certain operators deforming the Lagrangian). Let us briefly describe the general form of the string dual. The 'seed' background describes the backreaction of a set of N_c D5 branes wrapping a two-cycle inside the resolved conifold. It consists of a metric, dilaton $\phi(\rho)$ and RR three-form F_3 and, in string frame, it reads (the coordinates used are $[t, \vec{x}, \rho, \theta, \varphi, \tilde{\theta}, \tilde{\varphi}, \psi]$),

$$
\begin{aligned}
ds^2 &= e^{\phi(\rho)}\Big[dx_{1,3}^2 + e^{2k(\rho)}d\rho^2 + e^{2h(\rho)}(d\theta^2 + \sin^2\theta d\varphi^2) + \\
&+ \frac{e^{2g(\rho)}}{4}\left((\tilde{\omega}_1 + a(\rho)d\theta)^2 + (\tilde{\omega}_2 - a(\rho)\sin\theta d\varphi)^2\right) + \frac{e^{2k(\rho)}}{4}(\tilde{\omega}_3 + \cos\theta d\varphi)^2\Big], \\
F_{(3)} &= \frac{N_c}{4}\Big[-(\tilde{\omega}_1 + b(\rho)d\theta) \wedge (\tilde{\omega}_2 - b(\rho)\sin\theta d\varphi) \wedge (\tilde{\omega}_3 + \cos\theta d\varphi) + \\
&\quad b'd\rho \wedge (-d\theta \wedge \tilde{\omega}_1 + \sin\theta d\varphi \wedge \tilde{\omega}_2) + (1 - b(\rho)^2)\sin\theta d\theta \wedge d\varphi \wedge \tilde{\omega}_3\Big].
\end{aligned}
\tag{3.28}
$$

where $\tilde{\omega}_i$ are the left-invariant forms of $SU(2)$

$$
\begin{aligned}
\tilde{\omega}_1 &= \cos\psi d\tilde{\theta} + \sin\psi \sin\tilde{\theta}d\tilde{\varphi}, \quad \tilde{\omega}_2 = -\sin\psi d\tilde{\theta} + \cos\psi \sin\tilde{\theta}d\tilde{\varphi}, \\
\tilde{\omega}_3 &= d\psi + \cos\tilde{\theta}d\tilde{\varphi}.
\end{aligned}
\tag{3.29}
$$

This supersymmetric system was carefully studied in a series of papers [9], [10]; where it was shown that there is a combination of background functions (basically a 'change of basis') that move from a set of coupled BPS equations to a decoupled one that one can solve-up to one function, that we will call $P(\rho)$. We will not insist much with this formalism here and refer the interested reader to the original work [9], [10][5].

We will be more restrictive and for the purposes of this paper, we will study solutions where the functions $a(\rho) = b(\rho) = 0$ in the background of eq.(3.28). This is just in order to make our numerics simpler and illustrate the points we want to make. If we wanted to construct a black hole solution showing explicitly the transition between R-symmetry breaking and its restoration, we should then work without this restriction. Since in all known solutions the asymptotics of the functions $a(\rho) \sim b(\rho) \sim e^{-2\rho}$ our asymptotic results will be qualitatively correct, but near the black hole horizon there could be important differences.

We will then proceed as follows: first we will propose a background including a black hole with the restrictions mentioned above ($a(\rho) = b(\rho) = 0$). We will then pass it through the solution generating machine. This will produce a new background, now including also F_5, H_3, that we will write explicitly (we have checked that the equations of motion before and after the U-duality are the same). We will be explicit about the asymptotics of each of the functions

---

[5]All this formalism was also applied to the case in which one also adds fundamental matter to the dual QFT or sources to the background above. We refer the interested reader to the review [21].

near the UV and near the horizon. This will produce for us a non-extremal generalization of the type IIB string background dual to the 'baryonic branch of KS-system' -certainly with the restriction that our background does not contain the information of the breaking of the R-symmetry from $Z_{2N} \to Z_2$.

3.1   A new 'seed' solution

In this section we consider the background presented in equation (3.28) with the restrictions $a = b = 0$ but including the non-extremality factors. In string frame we have,

$$ds_{IIB,s}^2 = e^\phi \Big[-h(\rho)dt^2 + dx_1^2 + dx_2^2 + dx_3^2 \Big] + ds_{6,s}^2,$$

$$ds_6^2 = e^\phi \Big[\frac{e^{2k}}{s(\rho)}d\rho^2 + \frac{e^{2k}}{4}(\tilde{\omega}_3 + \cos\theta d\varphi)^2 + e^{2q}(d\theta^2 + \sin^2\theta d\varphi^2) + \frac{e^{2g}}{4}(d\tilde{\theta}^2 + \sin^2\tilde{\theta}d\tilde{\varphi}^2) \Big],$$

$$F_{(3)} = \frac{N_c}{4}\Big[-\tilde{\omega}_1 \wedge \tilde{\omega}_2 + \sin\theta d\theta \wedge d\varphi \Big] \wedge (\tilde{\omega}_3 + \cos\theta d\varphi) = \frac{\alpha' N_c}{4}w_3, \tag{3.30}$$

where $h(\rho), s(\rho)$ are the non-extremality functions. In the following, we will set $s(\rho) = h(\rho)$ which is simply a choice of parametrization; this implies that (on a particular solution) other background functions $\{q(\rho), g(\rho), k(\rho), \phi(\rho)\}$ need not take the same values as in the SUSY background.

The above Ansatz might be familiar to some readers. It is worth highlighting a key difference with previous work. Since our goal is to 'rotate' this solution we look for solutions with stabilized dilaton. Namely, the typical asymptotic value of the dilaton inherited from the D5 (or NS5 brane) solution is linear [32], more precisely [23] [25],

$$e^\phi \sim e^\rho \rho^{-1/4} \tag{3.31}$$

We are looking for a dilaton that stabilizes at infinity, that is, which asymptotically behaves as:

$$e^\phi \sim 1 + \mathcal{O}\left(e^{-8/3\rho}\right). \tag{3.32}$$

Our solution is qualitatively characterized by two parameters: one describing the nonextremality of the solution and the other the speed at which the dilaton gets stabilized.

After applying the solution generating technique, we end up with a new Type IIB background containing F_5, H_3 aside from the fields originally present in eq.(3.30) (all details are

written in the Appendix E),

$$ds^2_{IIB,st} = H^{-1/2}\Big[-hdt^2 + dx_1^2 + dx_2^2 + dx_3^2 \Big] + e^{2\phi}H^{1/2}\Big[e^{2k}\frac{d\rho^2}{h(\rho)} + \frac{e^{2k}}{4}(\tilde\omega_3 + \cos\theta d\varphi)^2 +$$

$$+ e^{2q}(d\theta^2 + \sin^2\theta d\varphi^2) + \frac{e^{2g}}{4}(d\tilde\theta^2 + \sin^2\tilde\theta d\tilde\varphi^2) \Big],$$

$$H^{1/2} = \cosh\beta\sqrt{e^{-2\phi} - h(\rho)\tanh^2\beta}, \quad \phi_{new} = \phi(\rho),$$

$$F_3 = (\frac{\alpha' N_c}{4})\cosh\beta w_3, \quad H_3 = -\sqrt{h}(\frac{\alpha' N_c}{4})\sinh\beta\, e^{2\phi} *_6 w_3$$

$$F_5 = -\tanh\beta(1 + *_{10})\mathrm{vol}_4 \wedge d(\frac{h}{H}). \tag{3.33}$$

In the following, we will focus in the limit $\beta \to \infty$, this is the field theory-limit where the warp factors vanish at infinity. We will then rescale

$$\tilde N = N\cosh\beta, \quad x_i \to \sqrt{\cosh\beta}\sqrt{\tilde N \alpha'}\, x. \tag{3.34}$$

With all the above the $\beta \to \infty$ limits are finite and the solution is given by

$$ds^2_{IIB,st} = \tilde N\alpha'\Big[\tilde H^{-1/2}(-hdt^2 + dx_i dx^i) + e^{2\phi}\tilde H^{1/2}ds_6^2 \Big],$$

$$F_3 = \frac{\alpha'\tilde N}{4}w_3, \quad H_3 = -\sqrt{h}\frac{\alpha'\tilde N}{4}e^{2\phi} *_6 w_3$$

$$F_5 = -(\alpha'\tilde N)^2(1 + *_{10})\mathrm{vol}_4 \wedge d(\frac{h}{\tilde H}) \tag{3.35}$$

with

$$\tilde H^{1/2} = \sqrt{e^{-2\phi} - h(\rho)} \tag{3.36}$$

It is worth making a few observations:

- We have two new classes of solutions. The equations of motion before and after the 'rotation' are the same, thus a solution of the system of equations (3.30) furnishes a solution of the form (3.33)-(3.35).

- The non-extremality factor have made its way into the RR five form and the warp factor in eq.(3.36), but the dilaton and the three forms are unchanged from the extremal case. Notice that the factor of $\sqrt{h}$ in the NS field H_3 is canceled by the self dual in six dimensions $*_6 w_3$. See Appendix E for full details.

- The black hole in the seed solution will likely have negative specific heat but its dilaton is stabilized which is a crucial difference with solutions considered previously. We will discuss what is the behavior of the solution after the rotation.

In the following section we will explicitly describe the asymptotic behavior of the solution as well as its numerical presentation in the background given by eq.(3.30).

3.2 Asymptotics

We will proceed to study the asymptotics of the equations of motion and solve them numerically to obtain the new non-extremal seed solution (3.30).

3.2.1 UV expansion

A large ρ expansion of the functions (k, q, g, h, ϕ), that solve the equations of motion up to terms decaying faster than $e^{-4\rho}$ is [6]

$$h(\rho) = e^{-8x(\rho)} \sim 1 + C_2 e^{-\frac{8}{3}\rho},$$

$$e^{2q(\rho)} \sim \frac{N_c}{4}(2\rho - 1) + \frac{c}{4}e^{\frac{4}{3}\rho}, \quad e^{2g(\rho)} \sim \frac{N_c}{4}(1 - 2\rho) + \frac{c}{4}e^{\frac{4}{3}\rho},$$

$$e^{2k(\rho)} \sim \frac{2c}{3}e^{\frac{4}{3}\rho} - \frac{N_c^2}{6c}(25 - 40\rho + 16\rho^2)e^{-\frac{4}{3}\rho}, \quad e^{4\phi - 4\phi_0} \sim 1 + \frac{3N_c^2}{4c^2}(1 - 8\rho)e^{-\frac{8}{3}\rho}. \quad (3.37)$$

There are two parameters in the expansion above. The parameter C_2 characterizes the non-extremal behavior. The parameter c plays a quite important role. It is one of the integration constants of the BPS eqs. As an expansion in inverse powers of this constant it is possible to write a solution for the background in eq.(3.28)-see for example [10], [22]-such that when passed through this solution generating technique, it results in the Baryonic Branch dual solution of [7]. Physically (and in the background before the rotation), the constant c is describing the coupling of the dynamics on the five-brane to gravity and ultimately to the full string theory, it takes the configuration out of its 'near brane' limit and can be thought of as indicating the insertion of an irrelevant operator in the QFT lagrangian.

3.2.2 Near Horizon Asymptotics

Near the horizon, $\rho = \rho_h$, we expand the equations of motion in power series up to order 7. We have found,

$$e^{-8x(\rho)} = x_1(\rho - \rho_h) + x_2(\rho - \rho_h)^2 + x_3(\rho - \rho_h)^3 + \cdots + x_8(\rho - \rho_h)^8$$

$$e^{2q(\rho)} = q_0 + q_1(\rho - \rho_h) + q_2(\rho - \rho_h)^2 + \cdots + q_7(\rho - \rho_h)^7$$

$$e^{2g(\rho)} = g_0 + h_1(\rho - \rho_h) + g_2(\rho - \rho_h)^2 + \cdots + g_7(\rho - \rho_h)^7 \qquad (3.38)$$

$$e^{2k(\rho)} = k_0 + k_1(\rho - \rho_h) + k_2(\rho - \rho_h)^2 + \cdots + k_7(\rho - \rho_h)^7$$

$$e^{4\Phi(\rho)} = f_0 + f_1(\rho - \rho_h) + f_2(\rho - \rho_h)^2 + \cdots + f_7(\rho - \rho_h)^7$$

$$(3.39)$$

Demanding that the expansions in eq.(3.38) satisfy the equations of motion of the system we determine $x_2, ..., h_1..., g_1..., k_1..., f_1...$ in terms of x_1, h_0, g_0, k_0, f_0 and thus, there are only 5

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[6]Note that the UV expansion for $h(\rho) = e^{-8x}$ could have started with a lower order term such as $e^{-2\rho}$, see Section 4.1 for a discussion on the choice of parameters.
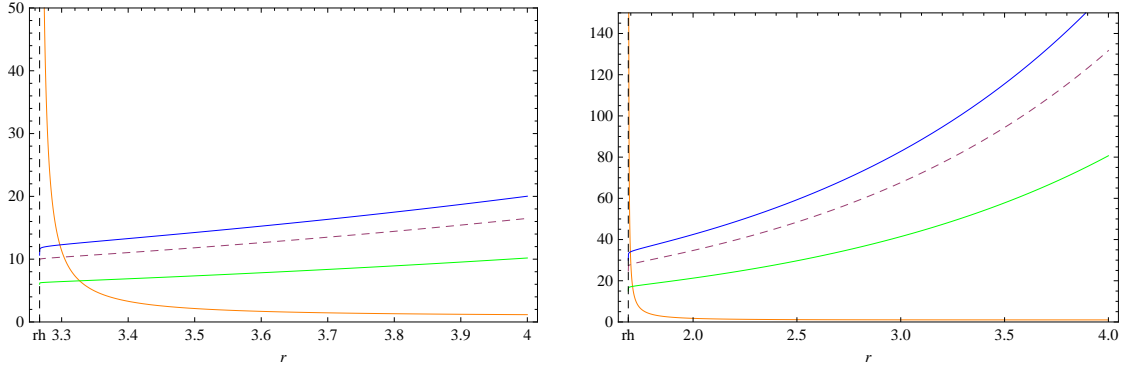
Figure 1: Solution before the rotation for $c = 1.97$, $C_2 = -7000$. The blue, dashed, green and orange lines represent e^q, e^k, e^g and h^{-1} respectively.

Figure 2: Solution before the rotation for $c = 125.62$, $C_2 = -89$. The blue, dashed, green ad orange lines represent e^q, e^k, e^g and h^{-1} respectively.

independent parameters coming from the expansion at the horizon. Furthermore, x_1 is related to the non-extremality parameter α-see Appendix F for the definitions,

$$\alpha = -\frac{x_1\sqrt{f_0}g_0h_0}{8} \tag{3.40}$$

3.3 Numerics

The strategy we will follow is to numerically integrate back from infinity the equations of motion in Appendix F, using as boundary conditions the expansion in the UV-eq.(3.37)-and require that at the horizon this numerical solution matches the expansion (3.38) and its first derivatives -up to seventh order. Doing this for a given set of values (c, C_2), determines the free parameters $(x_1, h_0, g_0, k_0, f_0)$. Note that not for every pair of (c, C_2) there is a black hole solution. For some values of (c, C_2) there will not be a horizon and for others there might be naked singularities outside the horizon. The study of the this two-parameter family of solutions for the full (c, C_2) parameter space is an interesting question that we will not address here. We concentrate on finding particular values of (c, C_2) that will produce black hole solutions. Needless to say, finding a solution before the rotation guarantees that we will have a solution after the rotation. Examples of numerical solutions are shown in Figure 1 and 2 and the respective dilatons in Figures 3 and 4.

3.3.1 Comments about the numerical method

As mentioned above we integrate back from infinity and match with the expansion at the horizon. In order to do so we define a "mismatch" function evaluated at a point ρ_0 close to
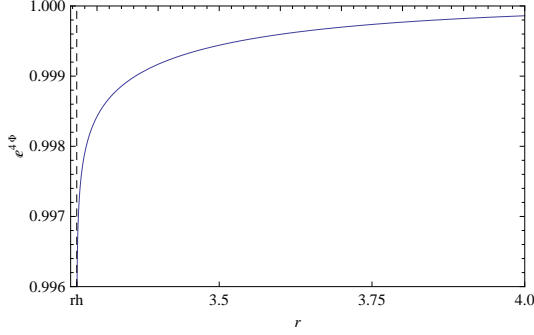
14

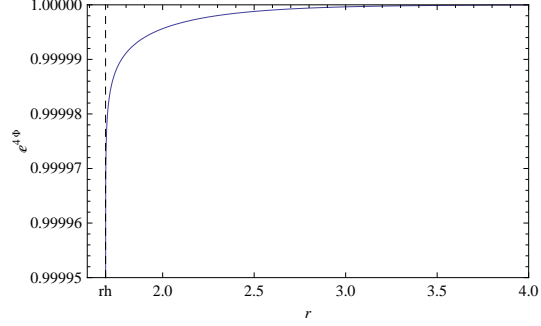Figure 3: Dilaton, $e^{4\Phi}$ for $c = 1.97, C_2 = -7000$.

Figure 4: Dilaton, $e^{4\Phi}$ for $c = 125.62, C_2 = -89$.

the horizon, $\rho_0 = \rho_h + 0.05$

$$m(\rho_0) = [g_{sh}(\rho_0) - g_{num}(\rho_0)]^2 + [g'_{sh}(\rho_0) - g'_{num}(\rho_0)]^2 + [h_{sh}(\rho_0) - h_{num}(\rho_0)]^2 + \\ [h'_{sh}(\rho_0) - h'_{num}(\rho_0)]^2 + [k_{sh}(\rho_0) - k_{num}(\rho_0)]^2 + [k'_{sh}(\rho_0) - k'_{num}(\rho_0)]^2 + \\ [\Phi_{sh}(\rho_0) - \Phi_{num}(\rho_0)]^2 + [\Phi'_{sh}(\rho_0) - \Phi'_{num}(\rho_0)]^2. \tag{3.41}$$

The subscript num denotes the numerical solution obtained by integrating back from infinity and sh denotes the series expansion (3.38) -evaluated at ρ_0. Recall that the expansion at the horizon depends on five free parameters, $(x_1, h_0, g_0, k_0, f_0)$. For a given set of (c, C_2) we minimize the mismatch function to determine $(x_1, h_0, g_0, k_0, f_0)$. The parameters at infinity (c, C_2) are adjusted so that $m(\rho_0) < 5 \times 10^{-5}$. All the numerical procedures where done in Mathematica using WorkingPrecission 40. We also checked that the constraint $T + U = 0$ (see appendix F) remains numerically negligible throughout the domain.

3.4 The rotated solution

Once we have obtained a numerical solution for the non-extremal background (3.30) we can easily generate the rotated solution as summarized in eq.(3.35) and outlined in the Appendix E. Before presenting the numerical results let us look at the large ρ asymptotics. Using the UV expansions of the seed solution (3.37), one can obtain the black hole metric (3.35) asymptotics

after the rotation,

$$-g_{tt} \sim \frac{e^{4/3\rho}}{A(\rho)} - \frac{e^{-4/3\rho}}{512c^4\sqrt{2}A(\rho)^3}[256c^4C_2^2 + 27(1 - 8\rho)^2 + 96c^2C_2(-1 + 8\rho)] + \cdots,$$

$$g_{xx} \sim \frac{e^{4/3\rho}}{A(\rho)} - \frac{e^{-4/3\rho}}{512c^4\sqrt{2}A(\rho)^3}[27(1 - 8\rho)^2] + \cdots,$$

$$g_{\rho\rho} \sim \frac{2c}{3}A(\rho) + \cdots, \quad g_{\theta\theta} \sim \frac{c}{4}A(\rho) + \frac{e^{-4/3\rho}}{4}A(\rho)(2\rho - 1) + \cdots,$$

$$g_{\psi\psi} \sim \frac{c}{6}A(\rho) + \cdots, \quad g_{\tilde{\theta}\tilde{\theta}} \sim \frac{c}{4}A(\rho) - \frac{e^{-4/3\rho}}{4}A(\rho)(2\rho - 1) + \cdots. \tag{3.42}$$

Where we have defined the quantity

$$c\sqrt{8}A(\rho) \equiv \sqrt{24\rho - 8C_2c^2 - 3}. \tag{3.43}$$

Using a variable $u = e^{2\rho/3}$ the metric can be written -to leading order- as,

$$ds^2 = \frac{u^2}{A(u)}(dx_i dx^i) + \frac{3cA(u)}{2}(\frac{du^2}{u^2} + ds_{T^{11}}^2) + \cdots \mathcal{O}(u^{-2}). \tag{3.44}$$

Thus, the non-extremal solution found after the rotation has the asymptotics of the Klebanov-Strassler background [8]. We point out that the reason we get precisely Klebanov-Tseytlin type of asymptotics lies in our precise arrangements of coefficients in the seed solution. Namely we arranged for a seed solution with no linear dilaton and with a dilaton behavior that, in the field theory limit, eliminates the leading term in the harmonic function given in equation (3.36).

With our careful choice of asymptotic conditions for the seed solution of D5 branes we have managed to construct black holes in asymptotically KT backgrounds. Our solution is different from those obtained in the traditional approach to black holes in asymptotically KT backgrounds (see, for example, [33]). One glaring difference is the form of the RR fluxes. In particular the presence of the non-extremality parameter in F_5 and in the warp factor $H(\rho)$ is completely novel.

In Figures 5-8 we present the numerical plots of the diffent metric elements after the rotation for two sets of parameters. In Figures 9 and 10 we see how the angular part of the solution approaches the $T^{1,1}$ metric for the two representative solutions.

3.5   Comments on thermodynamics

Let us clarify the expectations of the form of the background that arise from field theory considerations. In the previous section we obtained a solution largely characterized by two parameters (c, C_2). After the rotation, the interpretation of c has been spelled out explicitly
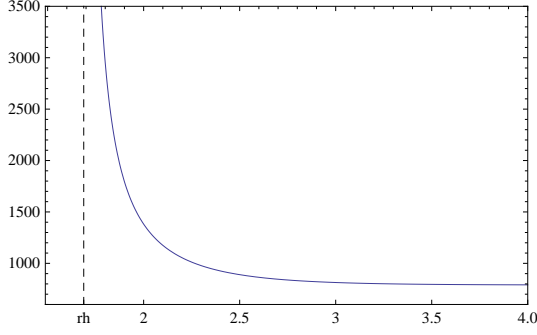
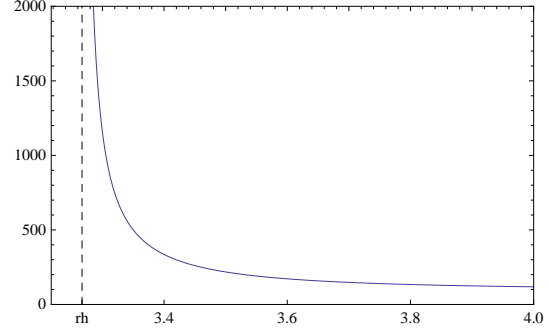Figure 5: The g_{rr} metric element for a solution after the rotation, $c = 125.62$, $C2 = -89$.

Figure 6: The g_{rr} metric element for a solution after the rotation, $c = 1.97$, $C_2 = -7000$.

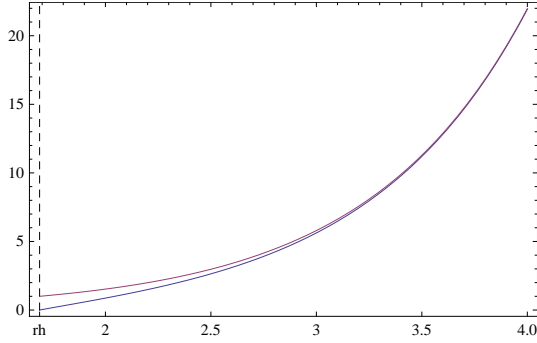

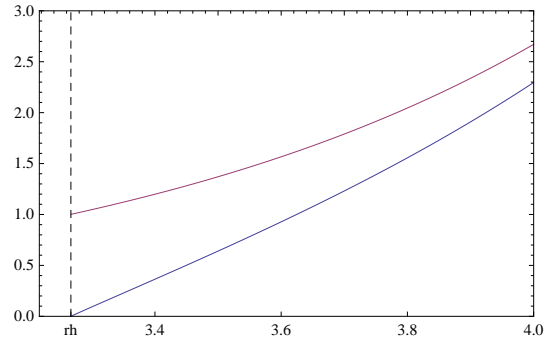Figure 7: g_{xx} and g_{tt} for a solution after the rotation, $c = 125.62$, $C_2 = -89$.

Figure 8: g_{xx} and g_{tt} for a solution after the rotation, $c = 1.97$, $C_2 = -7000$.

Figure 9: Angular part of the metric after the rotation. We plot $g_{\theta\theta}$, $g_{\tilde\theta\tilde\theta}$ and $\frac{3}{2}g_{\psi\psi}$. Solution with $c = 125, C_2 = -89$.
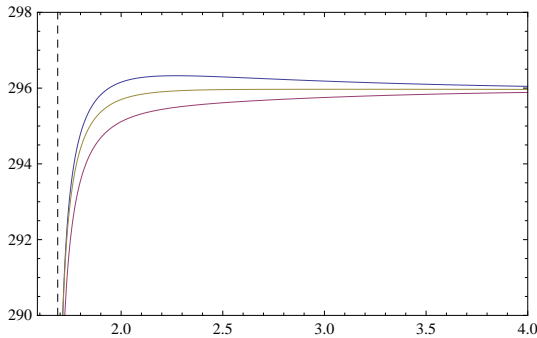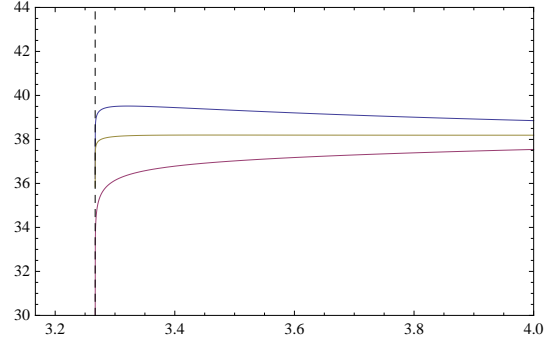
Figure 10: Angular part of the metric after the rotation. We plot $g_{\theta\theta}$, $g_{\tilde\theta\tilde\theta}$ and $\frac{3}{2}g_{\psi\psi}$. Solution with $c = 1.97, C_2 = -7000$.

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in [15], [6] and [13]. It is related to the vev of the baryon operator. The other parameter present in the solution is C_2, its interpretation coincides roughly with the temperature of the solution in the field theory.

We will present and exhaustive analysis of these classes of solutions elsewhere and will limit our attention here to a few main points. The main deterrent in presenting a final description of the thermodynamics is the need to explore a large parameter space of the solutions as well as the difficulty in matching data at asymptotic infinity with data near the horizon. The analysis is numerically costly but we emphasize that these are not difficulties of principle. Our goal in this work is to explicitly show, through numerical analysis, the existence of the conjectured solutions.

Let us show that the thermodynamic analysis can, in principle, be done explicitly for the solutions we have. For the temperature, we use the standard approach of looking at the Euclidean section and imposing the absence of conical singularities to determine the temperature of the supergravity backgrounds. Basically, for a metric with Euclidean section of the form

$$ds^2 = fd\tau^2 + \frac{d\rho^2}{g} + \ldots, \tag{3.45}$$

we find a temperature equal to

$$T = \frac{1}{4\pi}\sqrt{f'\,g'}, \tag{3.46}$$

where prime represents derivative with respect to the radial coordinate and the above expression is evaluated at the horizon. Interestingly, the temperature in both backgrounds, that is, before and after the rotation, is the same. In terms of the near horizon data we have

$$T = \frac{1}{4\pi}\frac{x_1}{\sqrt{k_0}}. \tag{3.47}$$

It is worth noting that this is the temperature at the horizon. In the case of asymptotically flat backgrounds (before the rotation) we have the option of computing the temperature as seen by an observer at infinity. We refer the reader to [23] for a detailed discussion of this procedure in the D5 or NS5 background. The entropy is computed, as usual, as the area of the horizon. The most complicated quantity to compute is the free energy which is required to compute, for example, the specific heat. For the free energy we need to evaluate the action. In the case of asymptotically flat solutions for D5/NS5 branes we can essentially follow [23]. A very preliminary analysis seems to confirm our intuition about the specific heat of such solutions. Namely, evaluation of the action suggests that before the rotation the solutions obtained have negative specific heat. A precise evaluation of the action after the rotation is more subtle as naive evaluation yields divergent terms. It is likely that a more accurate approach requires some sort of holographic renormalization or substraction along the lines of [33]. Preliminary evaluation suggests a positive specific heat for the rotated solutions [7].

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[7]Dario Martelli suggested that along the lines of the first two papers in [3], some thermodynamical quantities should be invariant under T-dualities, shifts of coordinates and boosts. See also [34].

4 About decoupling limits

It is worth discussing a bit about decoupling limits. Let us do this in the extremal case, the extension to the cases with non extremal factors follows from what we write here. The first point to emphasize is that when gravity modes are coupled to a generic D-brane configuration, the warp factors asymptote to a constant. An example of this is what happens to the flat Dp branes, with $H = 1 + (\frac{Q}{\rho})^{7-p}$.

We will compare warp factors in the examples studied in the previous sections. It is enough to focus on the expressions of the metric component g_{tt}. In all the examples studied we found

$$g_{tt} = \frac{4AB - C^2}{4\sqrt{B}}. \tag{4.48}$$

Indeed, in the case in which we rotate a flat Dp-brane, we have eq.(A.6) with A, B, C given in eq.(A.5). In the case of the rotation in $p - q$ directions the results for A, B, C are written in eq.(2.25) and in the case of the wrapped D5's are given in eq.(E.9). Again, for the point we wish to emphasize, we will focus on the extremal cases, when the rotation preserves $SO(1, p)$. So, in these cases we have[8],

$$g_{tt,(p-q)} = -\frac{1}{\cosh\beta\sqrt{H - \tanh^2\beta}}, \; g_{tt,p} = -\frac{\sqrt{H}}{\sqrt{H^2(\hat{A}\tanh\beta - 1)^2 - \tanh^2\beta}},$$

$$g_{tt,D5} = -\frac{1}{\cosh\beta\sqrt{e^{-2\phi} - \tanh^2\beta}} \tag{4.49}$$

The case of the rotation in p directions shows that the induced charge of D0 brane (after the first set of T-dualities) as shown by eq.(A.2), enters in the new warp factor via the one-form potential $A_1 = \hat{A}dt$. This makes the decoupling of the final configuration more unclear and typically, if we start with 'decoupled' Dp branes (that is $H \sim \rho^{p-7}$) we will generate a non-decoupled configuration, with an asymptotically constant warp factor as discussed around eq.(2.13). Of course, we can try to take the decoupling limit in the final configuration.

On the other hand the cases of the wrapped D5's and the rotation in $p - q$ directions (that share the fact that a D0 charge is induced by the boost only) are such that we *must* start with a non-decoupled configuration in order to avoid a generated background with a singularity or end of the space in the UV (zero warp factor at some finite value of the radial coordinate). Indeed, if $H = 1 + (\frac{Q}{\rho})^{7-p}$ we choose a particular value of the boost parameter $\beta \to \infty$, together with a particular choice of the value of $e^{\phi(\infty)} = 1$ in the D5's case) to obtain a background decoupled from gravity[9].

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[8]Gregory Giecold correctly pointed out that in the non-extremal cases the non-extremality factor $h(\rho)$ enters as expected, for example $g_{tt,D5} = -\frac{h}{\cosh\beta\sqrt{e^{-2\phi} - h\tanh^2\beta}}$. We thank him for this and various other correct comments.

[9] One of the interesting observations in [6], is that in the case of the D5's the solution described in the

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4.1 Choice of Parameters for $\mathcal{N} = 1$ Black Holes

Let us discuss in more detail the choice of parameters in the solution. This is a particularly sensitive question from the numerical and from the physical points of view. We note that *a priori* there are ten parameters in our solution as corresponds to a system of five second order ordinary differential equations. The constraint lowers this number by one. At this point the simplest route for us is to notice that we are looking for a particular solution that asymptotes to the extremal supersymmetric one found in [10],[13] (see, in particular, appendix B there). In that context most of our constants get fixed, as we require the same UV behavior; we use the expressions in that solution to fix six constants. Of the three constants left, one gets fixed by the asymptotic behavior of the non-extremality function, that is, we choose

$$h(\rho) \sim 1 - C_2 e^{-\frac{8}{3}\rho}. \tag{4.50}$$

The leading coefficient guarantees that we will have, after rotation, a solution with KT asymptotics. The key observation to be made is that, in principle, there is a mode allowed that we have explicitly set to zero. Namely, another solution to the equations of motion in the asymptotic regime could be

$$h(\rho) \sim 1 + C_1 e^{-2\rho} - C_2 e^{-\frac{8}{3}\rho} + \dots. \tag{4.51}$$

The above expression should be thought of schematically as the solution to a second order differential equation for $h(\rho)$ which is defined asymptotically by two parameters. This C_1 is one of the three constants that we fix and therefore we remain with two constants that we called (c, C_2). The parameter c has the same meaning discussed in [10],[13] and C_2 is related to the temperature. Thus, the condition that the solution after the rotation has KT asymptotics plus a condition on the non-extremality function determine the values of the integration constants.

It is worth noting that selecting the right constants in the D3/D5 picture is more cumbersome as there are more parameters in the generic D3/D5 approach to black holes with KT asymptotics. For example, our solution does not allow for a generic mode that will lead to a Freund-Rubin form for F_5. As has been repeatedly emphasized, our F_5 is related to the non-extremality function and deforms the typical term proportional to the volume 5-form. Let us explicitly visualize the difficulty in choosing the asymptotic expansion from the D3/D5 point of view in another example. The choice $C_1 = 0$ roughly corresponds to a dimension 3 operator in the D3/D5 picture. This can be seen easily by going to the radial coordinate $u = \exp(2\rho/3)$.

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papers [10], [22] is such that the dilaton asymptotes to a (tunable) constant and the subleading terms are precisely those needed to generate the warp factor of Klebanov-Strassler

5 Conclusions and Future Directions

We present here some concluding remarks together with some possible future projects to complement and refine the material in this paper.

In this work we have elaborated on the U-duality proposed in [6]. In that case, it was used to connect solutions describing wrapped D5 branes with the family of solutions describing the strong coupling dynamics of the baryonic branch of the Klebanov-Strassler field theory. In our case, we have used this U-duality to construct a new family of solutions describing the finite temperature phase of this baryonic branch. But before doing so, we explored the U-duality (that, following custom, we called rotation) in more generality, as a solution generating technique. We applied it to flat Dp branes in two different circumstances so that we appreciated the subtle differences making the choice of dualities in [6] so special.

Once we developed this technology, we applied it to rotate the Witten model for a Yang-Mills dual [17]. All this represents new material and it would be interesting to find an application for it, hopefully in the context of duals to Yang-Mills or QCD-like field theories.

In section 3, we discussed various new things: first a class of non-extremal solutions describing five branes wrapping a two-cycle inside the resolved conifold. The main feature of these non-extremal backgrounds is that the dilaton is asymptotically stabilized, which we interpreted as coupling the dual field theory to gravity modes. Then, we applied the U-duality to this solution, to find a new background of IIB that describes a non-extremal deformation of the Klebanov-Tseytlin solution (*decoupled* from gravity or stringy modes). We briefly commented on the associated thermodynamic properties of this new family of backgrounds. Finally, we discussed various aspects of this coupling/decoupling from gravity modes in section 4.

Let us now list a set of projects that complement the material presented here and that may improve the understanding of the topic:

- It is important to sort out the details of the thermodynamics *after* the rotation in the solutions presented in section 3. Indeed, as we indicated in section 3.5, this poses a numerical problem, but certainly does not amount to a conceptual obstruction. The result for quantities like the specific heat or the free energy will be of great interest. Notice that this presents a novel way to approach the finite temperature phase of the Klebanov-Strassler field theory. Comparison with the solutions of [38] would be interesting.

- More conceptually; it would be nice to better understand the role played by the D0 branes in this solution generating technique. Indeed, notice the differences between the examples analyzed in section 2 and the one in section 2.1, all due to the presence of D0 branes *before* the boost that translates into the different final results. In the same vein, it would be good to have a better understanding on the reasons for the equivalence between the U-duality explained here and the rotation of the J_2, Ω_3 forms as described

in [12], [13].

- It would be of very much interest to extend our non-extremal solution to a background of the form in eq.(3.28) where the restriction $a(\rho) = b(\rho) = 0$ is not applied. Indeed, finding such a non-extremal solution would, after the application of the U-duality, produce a solution dual to a non-SUSY KS-like field theory in the baryonic branch. It may prove interesting to compare this new solution with that presented in the papers [35]. The different nature of the non-SUSY deformation will certainly reflect in different field theory aspects.

- It would be nice to use the solution presented in section 3 as a base to produce a non-SUSY deformation along the line proposed recently in [36]. The authors of [36] are finding SUSY breaking backgrounds by adding anti-D3 branes. Their solutions have singularities (and the problem is if such a singularity is physically acceptable). Proceeding as with the solution in this paper may set us on a different branch of solutions.

- It would be interesting to find a Physics application of the new backgrounds of section 2.1.1.

- It would be nice to extend the treatment here to the case in which the seed background contains $N_f \sim N_c$ flavors. This would be following the lead of the papers [24], [25]. The subtle point would be how to rotate such solutions (in other words, how the sources get represented in M-theory, see [37]. Comparison with the solutions of [38] would be interesting)

- We have presented two numerical solutions as examples of the type of backgrounds obtained. These examples correspond to specific values of the UV parameteers. We did not attempt a study of the full family of solutions in the whole parameter space. This is an interesting question worth investigating.

We hope to address some of these problems in the near future.

Acknowledgments:

A Appendix: U-duality in the case of flat Dp-branes

In this appendix we will give a sketchy derivation of the U-duality chain, in the case in which we rotate flat Dp-branes. We will not consider the non-extremal case, this can be done easily and the result of doing so is included in the first section of the paper. Here we want just to emphasize some points.

We start from the SUSY configuration in type II supergravity (string frame),

$$ds^2 = H^{-1/2}(-dt^2 + dx_p^2) + H^{1/2}(d\rho^2 + \rho^2 d\Omega_{8-p}),$$

$$e^{2\phi} = H^{\frac{3-p}{2}}, \ F_{p+2} = \partial_\rho \hat{A} dt \wedge dx_1 \wedge ... dx_p \wedge d\rho \tag{A.1}$$

where $\hat{A} = -A$ in the notation of Section 2. Following standard convention we define the charge of F_{8-p} as positive. The sign of F_{p+2} will then depend on whether we consider Minkowski or Euclidean worldvolume on the D_p brane. We apply T-duality in the x_p directions,

$$ds^2 = -H^{-1/2}dt^2 + H^{1/2}(dx_p^2 + d\rho^2 + \rho^2 d\Omega_{8-p}),$$

$$e^{2\phi} = H^{\frac{3}{2}}, \ F_2 = \partial_\rho \hat{A} dt \wedge d\rho \tag{A.2}$$

We lift this to M-theory, using $e^{\frac{4\phi}{3}} = H$,

$$ds_{11}^2 = H(dx_{11} + \hat{A} dt)^2 - H^{-1}dt^2 + dM_9, \ \ dM_9 = (dx_p^2 + d\rho^2 + \rho^2 d\Omega_{8-p}) \tag{A.3}$$

we now boost in the $t - x_{11}$ direction with rapidity β

$$dt \rightarrow \cosh\beta dt - \sinh\beta dx_{11}, \ \ dx_{11} \rightarrow -\sinh\beta dt + \cosh\beta dx_{11}, \tag{A.4}$$

to get

$$ds_{11}^2 = A dt^2 + B dx_{11}^2 + C dt dx_{11} + dM_9,$$
$$A = H^{-1}[H^2(\hat{A}\cosh\beta - \sinh\beta)^2 - \cosh^2\beta]$$
$$B = H^{-1}[H^2(\hat{A}\sinh\beta - \cosh\beta)^2 - \sinh^2\beta]$$
$$C = -2H^{-1}[H^2(\hat{A}\cosh\beta - \sinh\beta)(\hat{A}\sinh\beta - \cosh\beta) - \cosh\beta\sinh\beta] \tag{A.5}$$

and we prepare the metric for reduction back to IIA as

$$ds_{11}^2 = B(dx_{11} - a_t dt)^2 + B^{-1/2}\Big[g_{tt}dt^2 + B^{1/2}dM_9\Big],$$

$$a_t = -\frac{C}{2B}, \ \ g_{tt} = \frac{4AB - C^2}{4\sqrt{B}} \tag{A.6}$$

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we now write the IIA configuration as,

$$ds_{IIA}^2 = g_{tt}dt^2 + B^{1/2}(dx_p^2 + d\rho^2 + \rho^2 d\Omega_{8-p}), \ e^{2\phi} = B^{3/2}, \ F_2 = \partial_\rho a_t dt \wedge d\rho \quad \text{(A.7)}$$

and we T-dualize back in the x_p directions to get

$$ds_{II}^2 = g_{tt}dt^2 + B^{-1/2}dx_p^2 + B^{1/2}(d\rho^2 + \rho^2 d\Omega_{8-p}),$$

$$e^{2\phi} = B^{\frac{3-p}{2}}, \quad F_{p+2} = \partial_\rho a_t dt \wedge d\rho \wedge dx_1 .... \wedge dx_p \quad \text{(A.8)}$$

We see that the $SO(1,p)$ invariance is preserved, as $g_{tt}\sqrt{B} = -1$.

B Appendix: Black Dp branes

In this appendix we gather some useful facts about black Dp-brane solutions [26],[27],[28]. The metric in string frame is,

$$ds^2 = -\frac{f_+(r)}{\sqrt{f_-(r)}}dt^2 + \sqrt{f_-(r)}\sum_{i=1}^p dx^i dx^i + \frac{f_-(r)^{-\frac{1}{2}-\frac{5-p}{7-p}}}{f_+(r)}dr^2 + r^2 f_-(r)^{\frac{1}{2}-\frac{5-p}{7-p}}d\Omega_{8-p}^2 \quad \text{(B.1)}$$

where

$$f_\pm(r) = 1 - \left(\frac{r_\pm}{r}\right)^{7-p} \quad \text{(B.2)}$$

The mass per unit volume M and the R-R charge N are

$$M = \frac{1}{(7-p)(2\pi)^7 d_p l_P^8}((8-p)r_+^{7-p} - r_-^{7-p}), \quad N = c_p(r_+ r_-)^{\frac{7-p}{2}}, \quad \text{(B.3)}$$

where l_P is the ten dimensional Planck length $l_P = g_s^{1/4}l_s$ and $c_p, \ d_p$ are numerical factors,

$$d_p = 2^{5-p}\pi^{\frac{5-p}{2}}\Gamma\left(\frac{7-p}{2}\right)$$

$$c_p = \frac{1}{d_p g_s l_s^{7-p}} = \frac{(7-p)Vol_{S^{8-p}}}{(2\pi l_s)^{7-p}g_s} \quad \text{(B.4)}$$

The background also has a nontrivial dilaton and a R-R flux,

$$e^{-2\Phi} = g_s^{-2}f_-(r)^{-\frac{p-3}{2}}$$

$$\frac{1}{(2\pi l_s)^{7-p}}\int_{S^{8-p}} *F_{p+2} = N. \quad \text{(B.5)}$$

The Einstein frame metric ($g_{E\mu\nu} = \sqrt{g_s e^{-\Phi}}g_{st\mu\nu}$) has a horizon at $r = r_+$ and, for $p \leq 6$, a singularity at $r = r_-$. If $r_+ > r_-$ the singularity is covered by the horizon and the solution is a black hole. In the extremal case, $r_+ = r_-$, the space is singular except for $p = 3$.

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Let us define a new coordinate,

$$\rho^{7-p} = r^{7-p} - r_-^{7-p}$$

. This change of variables transforms (B.1) to a more familiar form,

$$ds^2 = H(\rho)^{-\frac{1}{2}}\left(-h(\rho)dt^2 + \sum_{i=1}^{p} dx^i dx^i\right) + H(\rho)^{\frac{1}{2}}\left(\frac{d\rho^2}{h(\rho)} + \rho^2 d\Omega_{8-p}^2\right) \tag{B.6}$$

where

$$H(\rho) = 1 + \frac{r_-^{7-p}}{\rho^{7-p}}$$

$$h(\rho) = 1 - \frac{r_+^{7-p} - r_-^{7-p}}{\rho^{7-p}} \tag{B.7}$$

Throughout the paper we use the notation,

$$L_p^{7-p} \equiv r_-^{7-p} \quad \text{and} \quad R_T^{7-p} \equiv r_+^{7-p} - r_-^{7-p}.$$

Note that using (B.3) one can also write

$$L_p^{7-p} = \tilde{Q}\left[\sqrt{1 + \left(\frac{R_T^{7-p}}{2\tilde{Q}}\right)^2} - \frac{R_T^{7-p}}{2\tilde{Q}}\right]$$

$$= \tilde{Q}\,\alpha_p. \tag{B.8}$$

where $\tilde{Q} = \frac{N}{c_p}$ and $\alpha_p = (1 + (\frac{R_T^{7-p}}{2\tilde{Q}})^2)^{1/2} - \frac{R_T^{7-p}}{2\tilde{Q}}$.

¿From (B.3) we can see that the parameter R_T is related to the energy per unit volume above extremality ϵ,

$$\Delta M = \epsilon = \frac{r_+^{7-p} - r_-^{7-p}}{(2\pi)^7 d_p l_P^8} = \frac{R_T^{7-p}}{(2\pi)^7 d_p l_P^8}. \tag{B.9}$$

Note that in the decoupling limit ($\alpha' \to 0$, energies fixed) ϵ remains fixed and $\alpha_p \to 1$.

In the extremal case, $r_+ = r_-$, we have $R_T = 0$ and $L_p^{7-p} = \tilde{Q} = N d_p g_s l_s^{7-p}$.

C   Appendix: Some details on the U-duality

Here we provide details for the derivations of eq.(2.24). We started with

$$ds^2 = H^{-1/2}[-dt^2 + dx_{p-q}^2 + d\sigma_q^2] + H^{1/2}[d\rho^2 + \rho^2 d\Omega_{8-p}^2],$$

$$F_{p+2} = \partial_\rho \hat{A} dt \wedge dx_1 \wedge ... \wedge dx_{p-q} \wedge d\sigma_q \wedge d\rho,$$

$$e^{2\phi[initial]} = e^{2\phi(\infty)} H^{\frac{3-p}{2}}. \tag{C.1}$$

The we perform the U-duality described in the text below eq.(2.23)

$$
\begin{aligned}
ds^2 &= H^{-1/2}[-dt^2 + d\sigma_q^2] + H^{1/2}[d\rho^2 + \rho^2 d\Omega_{8-p}^2 + dx_{p-q}^2], \\
F_{q+2} &= \partial_r \hat{A} dt \wedge d\sigma_q \wedge d\rho, \\
e^{2\phi} &= e^{2\phi(\infty)} H^{\frac{3-q}{2}}.
\end{aligned}
\tag{C.2}
$$

Now, we need this to be a configuration in Type IIA (in order to lift this to M-theory). We will also want to impose that when lifted to M-theory this will produce a four-form field G_4. If this is the case, we must have that either $q = 2$ or that $q = 4$ (the cases of $q = 0, q = 6$ are analogous to what we analyzed in the first section). The case $q = 2$, on which we will elaborate upon below, has the peculiarity that the reduction from eleven dimensions back to IIA will generate a NS H_3 field, proportional to the boost. Let us see this in detail. To begin with, we will lift to eleven dimensions the configuration in eq.(C.2).

$$
\begin{aligned}
ds_{11}^2 &= H^{1/3} dx_{11}^2 + H^{-2/3}[-dt^2 + d\vec{\sigma}_2^2] + H^{1/3}[d\rho^2 + \rho^2 d\Omega_{8-p}^2 + d\vec{x}_{p-2}^2], \\
G_4 &= \partial_\rho \hat{A} dt \wedge d\rho \wedge d\sigma_1 \wedge d\sigma_2
\end{aligned}
\tag{C.3}
$$

and now we boost with rapidity β

$$
dt \to \cosh\beta \, dt - \sinh\beta \, dx_{11}, \quad dx_{11} \to -\sinh\beta \, dt + \cosh\beta \, dx_{11}
\tag{C.4}
$$

and we can rewrite the configuration after the boost as,

$$
\begin{aligned}
ds_{11}^2 &= A dt^2 + B dx_{11}^2 + C dt dx_{11} + dM_9^2, \\
G_4 &= \partial_\rho \hat{A}(\cosh\beta \, dt - \sinh\beta \, dx_{11}) \wedge d\rho \wedge d\sigma_1 \wedge d\sigma_2, \\
dM_9^2 &= H^{-2/3} d\vec{\sigma}_2^2 + H^{1/3}[d\rho^2 + \rho^2 d\Omega_{8-p}^2 + d\vec{x}_{p-2}^2], \\
A &= H^{-2/3}[H \sinh^2\beta - \cosh^2\beta], \\
B &= H^{-2/3}[H \cosh^2\beta - \sinh^2\beta], \\
C &= 2H^{-2/3} \sinh\beta \cosh\beta (1 - H)
\end{aligned}
\tag{C.5}
$$

when we reduce this to IIA we get,

$$
\begin{aligned}
ds_{IIA,st}^2 &= g_{tt} dt^2 + B^{1/2}\left(H^{-2/3} d\vec{\sigma}_2^2 + H^{1/3}[d\rho^2 + \rho^2 d\Omega_{8-p}^2 + d\vec{x}_{p-2}^2]\right), \\
e^{2\phi[A]} &= B^{3/2}, \quad g_{tt} = \frac{4AB - C^2}{4\sqrt{B}}, \\
F_4 &= \partial_\rho \hat{A}(\cosh\beta + a_t \sinh\beta) dt \wedge d\rho \wedge d\sigma_1 \wedge d\sigma_2, \\
H_3 &= \sinh\beta \, \partial_\rho \hat{A} d\rho \wedge d\sigma_1 \wedge d\sigma_2, \\
F_2 &= \partial_\rho(a_t) dt \wedge d\rho, \quad a_t = \frac{C}{2B} = \frac{\sinh\beta \cosh\beta (1 - H)}{H \cosh^2\beta - \sinh^2\beta},
\end{aligned}
\tag{C.6}
$$

we see that we have generated a NS magnetic field. Finally, we T-dualize back in the $\vec{x}_{p-1}$ directions, to get

$$ds_{II,st}^2 = g_{tt}dt^2 + \frac{d\vec{x}_{p-2}^2}{H^{1/3}B^{1/2}} + B^{1/2}\Big(H^{-2/3}d\vec{\sigma}_2^2 + H^{1/3}[d\rho^2 + \rho^2 d\Omega_{8-p}^2]\Big),$$
$$F_{p+2} = \partial_\rho\hat{A}(\cosh\beta + a_t\sinh\beta) \wedge d\rho \wedge d\sigma_1 \wedge d\sigma_2 \wedge dx_1 \wedge .... \wedge dx_{p-2},$$
$$H_3 = \sinh\beta\partial_\rho\hat{A}d\rho \wedge d\sigma_1 \wedge d\sigma_2,$$
$$F_p = \partial_\rho(a_t)dt \wedge d\rho \wedge dx_1 \wedge .... \wedge dx_{p-2},$$
$$e^{2\phi[final]} = B^{\frac{5-p}{2}}H^{\frac{2-p}{3}}. \tag{C.7}$$

This completes our derivation of eq.(2.24).

D   Appendix: Another solution generating algorithm

We studied two different 'solution generating techniques' and applied them to different backgrounds. All these 'algorithms' were starting with a background solution to the Type II (A or B) equations of motion, applying a number of T-dualities that would transform the background into a solution for Type IIA supergravity. Then we lifted this to eleven dimensions, where a boost was applied (inducing a one parameter-β- family of solutions), then reducing to IIA and T-dualizing back we had our final generated background.

One may wonder what is the algorithm when, after a number of T-dualities, we end with a background solving the Type IIB supergravity equations of motion. In this case, a way of generating a one-parameter family of solutions may be S-dualizing. Indeed, given a, b, c, d real numbers satisfying $ad - bc = 1$, we can start with a IIB solution having axion χ, dilaton ϕ, RR and NS thre forms $F_3 = dC_2$ and $H_3 = dB_2$ and five form $F_5 = dC_4 + F_3 \wedge B_2$ and by S-duality, generate a new solutions dependent on the three independent real parameters a, b, c. Indeed, the five form is left invariant and the same happens for the Einstein frame metric, while

$$F_3[new] = bH_3 + aF_3, \quad H_3[new] = dH_3 + cF_3,$$
$$e^{\phi[new]} = \Big((c\chi + d)^2 + c^2e^{-2\phi}\Big)e^\phi, \quad \chi[new] = \frac{(a\chi + b)(c\chi + d) + ace^{-2\phi}}{\Big((c\chi + d)^2 + c^2e^{-2\phi}\Big)} \tag{D.1}$$

Let us focus our attention in the particular set of values $a = d = 0$, $cb = -1$, this is the transformation that interchanges the three forms and inverts the value of the dilaton (if the axion is initially zero, as we will assume).

One can then think about applying this transformation expecting to generate new interesting solutions. As an example, suppose that we start with solutions describing D6 branes wrapping a three-cycle inside the deformed conifold. Those solutions are dual to a (UV completed version of) N=1 SYM [29]. A particularly interesting solution is given in section 3.2 of

the paper [30]. The background consist of metric, dilaton and RR two-form field,

$$ds^2_{IIA,st} = e^f[-dt^2 + dx_1^2 + dx_2^2 + dx_3^2 + dM_6^2], \quad e^{2\phi} = e^{3f+2\phi_0}, \quad F_2 = dA_1 \qquad (D.2)$$

where all the details (the manifold M_6, the functional form of f, ϕ and the one form A_1 are given in eqs(57)-(59) of [30]). We can the perform three T-dualities in $x_{1,2,3}$ leading to

$$ds^2_{IIB,st} = e^f[-dt^2 + dM_6^2] + e^{-f}dx^2_{1,2,3}, \quad e^{2\phi} = e^{2\phi_0},$$
$$F_5 = dA_1 \wedge dx_1 \wedge dx_2 \wedge dx_3 \qquad (D.3)$$

that is we have generated D3 brane charge and the dilaton is just a constant. We now move this to Einstein frame, that leaves us-up to a constant-with the same metric, perform the S-duality metioned above and T-dualize back. This brings us to the starting point background (D.2)[10]. Something similar occurs if we start with a solution describing D5 branes wrapping a three-cycle inside a G2 holonomy manifold [31], a configuration dual to N=1 Yang-Mills Chern-Simons in 2+1 dimensions (with its respective UV completion). Notice that this operation is not the one proposed in [20], that is the reason why these authors were able to generate an interesting solution starting from [31].

In both these cases described above, we are generating charge of D3 brane, this is invariant under the S-duality, hence it is expected that the whole operation brings us back to the initial configuration. A more interesting example is to start from the configuration of D4 branes wrapping a circle with SUSY breaking boundary conditions [17] that we discussed before, see Section 2.1.1.

In this case we will apply three T-dualities to a IIA configuration. This will generate charge of D1 brane that the S-duality will interchange with F1 charge, after T-dualizing back, we will generate a new background (the $SO(1,3)$ isometry will be spoiled, so we may want to start with the high Temperature dual, hence having a non-extremal factor $h(\rho)$ in front of dt^2 and the function $f(\rho) = 1$ in front of dx_4^2, but let us keep things general). Let us see some details. After the T-dualities, the configuration reads

$$ds^2_{IIB,st} = H^{-1/2}(-hdt^2 + fdx_4^2) + H^{1/2}(\frac{d\rho^2}{h} + \rho^2 d\Omega_4^2 + dx^2_{1,2,3}),$$
$$e^{2\phi} = g_s^2 H, \quad F_3 = \partial_\rho A dt \wedge dx_4 \wedge d\rho \qquad (D.4)$$

now, we need to move this to Einstein frame, multiplying the metric by $e^{-\frac{\phi}{2}}$, perform the S-duality that will generate H_3 and a dilaton $e^{-2\phi} = g_s^2 H$. Then, T-dualize back in $x_{1,2,3}$. The final configurations is,

$$ds^2_{IIA,st} = \frac{c^{1/4}}{g_s H}(-hdt^2 + fdx_4^2) + \frac{d\rho^2}{h} + \rho^2 d\Omega_4^2 + dx^2_{1,2,3},$$
$$H_3 = c\partial_\rho A dt \wedge dx_4 \wedge d\rho, \quad c^4 e^{-2\phi} = g_s^2 H. \qquad (D.5)$$

---

[10]One may perform an S-duality with parameters $a = 0, bc = -1$ and d being free. This will generate a background in IIB with constant axion-dilaton. The final IIA configuration is the same as the initial one.

As a final remark; had we chosen to T-dualize only in the x_4 direction (in the supergravity approximation, this can be done in the high Temperature phase only), we would have generated D3 branes, hence after the S-duality and T-duality, we would be back to the initial configuration.

E Appendix: U-duality for the wrapped D5 branes Black Hole

We will describe in detail the action of the solution generating technique proposed in [6], when applied to the background of eq.(3.30). These techniques have been applied in a similar context in [39]. Let us consider things in the string frame.

$$ds_s^2 = e^{\phi}\left[-h(\rho)dt^2 + dx_1^2 + dx_2^2 + dx_3^2\right] + ds_{6,s}^2$$

$$ds_6^2 = e^{\phi}\left[\frac{e^{2k}}{s(\rho)}d\rho^2 + \frac{e^{2k}}{4}(\tilde{\omega}_3 + \cos\theta d\varphi)^2 + e^{2q}(d\theta^2 + \sin^2\theta d\varphi^2) + \frac{e^{2g}}{4}(d\tilde{\theta}^2 + \sin^2\tilde{\theta}d\tilde{\varphi}^2)\right],$$

$$F_{(3)} = \frac{N_c}{4}\left[-\tilde{\omega}_1 \wedge \tilde{\omega}_2 + \sin\theta d\theta \wedge d\varphi\right] \wedge (\tilde{\omega}_3 + \cos\theta d\varphi), \tag{E.1}$$

where $h(\rho), s(\rho)$ are the non-extremality functions and we will choose $h(\rho) = s(\rho)$ as we did in section 3. We can proceed to rotate it. We will follow the procedure explained in [6]. So, let us start by writing the effect of the first T-duality in the x_1 direction (all the expressions below are in string frame)

$$ds_{IIA}^2 = e^f\left[-hdt^2 + dx_2^2 + dx_3^2\right] + e^{-f}dx_1^2 + ds_6^2,$$

$$e^{2\phi_A} = e^{2\phi-f}, \quad F_4 = F_3 \wedge dx_1 \tag{E.2}$$

The function $f = \phi$ will be kept to avoid confusion with the transformed dilatons. Now, we perform the T-duality in x_2

$$ds_{IIA}^B = e^f\left[-hdt^2 + dx_3^2\right] + e^{-f}(dx_1^2 + dx_2^2) + ds_6^2,$$

$$e^{2\phi_B} = e^{2\phi-2f}, \quad F_5 = F_3 \wedge dx_1 \wedge dx_2(1 + *_{10}) \tag{E.3}$$

T-dualizing in x_3, we get

$$ds_{IIA}^2 = e^f\left[-hdt^2\right] + e^{-f}(dx_1^2 + dx_2^2 + dx_3^2) + ds_6^2,$$

$$e^{2\phi_A} = e^{2\phi-3f},$$

$$F_6 = F_3 \wedge dx_1 \wedge dx_2 \wedge dx_3 \rightarrow F_4 = e^{2f}\sqrt{h} *_6 F_3 \wedge dt \tag{E.4}$$

Notice that

$$*_6 F_3 = \frac{N_c}{\sqrt{h}}\Big[-2e^{2q-2g} \sin\theta d\theta \wedge d\varphi + \frac{e^{2g-2q}}{8} \sin\tilde{\theta} d\tilde{\theta} \wedge d\tilde{\varphi} \Big] \wedge d\rho. \tag{E.5}$$

The factor of $\sqrt{h}$ in the F_4 of eq.(E.4) is present to cancel the factor of $\sqrt{h}$ in the denominator of eq.(E.5). Now, we lift this to M-theory;

$$ds_{11}^2 = e^{4/3\phi - 2f} dx_{11}^2 + e^{f-2/3\phi}\Big[-he^f dt^2 + e^{-f}(dx_1^2 + dx_2^2 + dx_3^2) + ds_6^2 \Big],$$
$$G_4 = \sqrt{h}e^{2f} *_6 F_3 \wedge dt. \tag{E.6}$$

We boost in the $t - x_{11}$ directions according to,

$$dt \to \cosh\beta dt - \sinh\beta dx_{11}, \quad dx_{11} \to -\sinh\beta dt + \cosh\beta dx_{11} \tag{E.7}$$

and now we rewrite this boosted metric as,

$$ds_{11}^2 = e^{f-2/3\phi}\Big[e^{-f}(dx_1^2 + dx_2^2 + dx_3^2) + ds_6^2 \Big] + Adt^2 + Bdx_{11}^2 + Cdtdx_{11},$$
$$G_4 = \sqrt{h}e^{2f} *_6 F_3 \Big[\cosh\beta dt - \sinh\beta dx_{11} \Big] \tag{E.8}$$

where,

$$A = e^{2f-2/3\phi}[\sinh^2\beta e^{2\phi-4f} - h\cosh^2\beta], \quad B = e^{2f-2/3\phi}[\cosh^2\beta e^{2\phi-4f} - h\sinh^2\beta],$$
$$C = -2\cosh\beta \sinh\beta e^{2f-2/3\phi}[e^{2\phi-4f} - h]. \tag{E.9}$$

Now, we will reduce this to IIA, before doing so and in order to reduce to IIA, it is useful to rewrite eq.(E.8) as,

$$ds_{11}^2 = B^{-1/2}\Big[g_{tt}dt^2 + B^{1/2}e^{f-2/3\phi}(e^{-f}(dx_1^2 + dx_2^2 + dx_3^2) + ds_6^2) \Big] + B(dx_{11} + a_t dt)^2,$$
$$G_4 = \sqrt{h}e^{2f} *_6 F_3 \Big[(\cosh\beta + a_t \sinh\beta)dt - \sinh\beta(dx_{11} + a_t dt) \Big] \tag{E.10}$$

where we have defined

$$a_t = \frac{C}{2B}, \quad g_{tt} = \frac{4AB - C^2}{4\sqrt{B}}, \quad e^{4/3\phi_A} = B. \tag{E.11}$$

Now, we reduce to IIA, obtaining in string frame,

$$ds_{IIA}^2 = g_{tt}dt^2 + \sqrt{B}e^{-2/3\phi}(dx_1^2 + dx_2^2 + dx_3^2) + \sqrt{B}e^{f-2/3\phi}ds_6^2,$$
$$e^{2\phi_A} = B^{3/2},$$
$$F_4 = \sqrt{h}e^{2f} *_6 F_3 \wedge \Big[(\cosh\beta + a_t \sinh\beta)dt \Big],$$
$$H_3 = -\sinh\beta\sqrt{h}e^{2f} *_6 F_3,$$
$$F_2 = a_t' d\rho \wedge dt \tag{E.12}$$

Now, we proceed to do the T-dualities back; T-dualizing in the x_3 direction we have

$$ds^2_{IIB} = g_{tt}dt^2 + \sqrt{B}e^{-2/3\phi}(dx_1^2 + dx_2^2) + \frac{e^{2/3\phi}}{\sqrt{B}}dx_3^2 + \sqrt{B}e^{f-2/3\phi}ds_6^2,$$

$$e^{2\phi_B} = Be^{2/3\phi},$$

$$F_5 = \sqrt{h}e^{2f} *_6 F_3 \wedge \left[(\cosh\beta + a_t\sinh\beta)dt \wedge dx_3\right](1 + *_{10}),$$

$$H_3 = -\sinh\beta\sqrt{h}e^{2f} *_6 F_3,$$

$$F_3 = a_t'd\rho \wedge dt \wedge dx_3 \tag{E.13}$$

now, we T-dualize in x_2

$$ds^2_{IIA} = g_{tt}dt^2 + \sqrt{B}e^{-2/3\phi}(dx_1^2) + \frac{e^{2/3\phi}}{\sqrt{B}}(dx_3^2 + dx_2^2) + \sqrt{B}e^{f-2/3\phi}ds_6^2,$$

$$e^{2\phi_A} = \sqrt{B}e^{4/3\phi},$$

$$F_6 = \sqrt{h}e^{2f} *_6 F_3 \wedge \left[(\cosh\beta + a_t\sinh\beta)dt\right] \wedge dx_3 \wedge dx_2,$$

$$H_3 = -\sinh\beta\sqrt{h}e^{2f} *_6 F_3,$$

$$F_4 = a_t'd\rho \wedge dt \wedge dx_3 \wedge dx_2 \tag{E.14}$$

finally, we T-dualize in x_1

$$ds^2_{IIB} = g_{tt}dt^2 + \frac{e^{2/3\phi}}{\sqrt{B}}(dx_3^2 + dx_1^2 + dx_2^2) + \sqrt{B}e^{f-2/3\phi}ds_6^2,$$

$$e^{2\phi_B} = e^{2\phi},$$

$$F_7 = \sqrt{h}e^{2f} *_6 F_3 \wedge \left[(\cosh\beta + a_t\sinh\beta)dt\right] \wedge dx_3 \wedge dx_2 \wedge dx_1 \qquad F_3 = *_{10}F_7,$$

$$H_3 = -\sinh\beta\sqrt{h}e^{2f} *_6 F_3,$$

$$F_5 = a_t'd\rho \wedge dt \wedge dx_3 \wedge dx_2 \wedge dx_1(1 + *_{10}). \tag{E.15}$$

After using $f = \phi$ and the definitions for A, B, C, a_t this encodes the result of eq.(3.35).

F   Appendix: The equations of motion

In this appendix we will quote the equations of motion that we are numerically solving. Our goal is to find a black hole of the metrics described in the main text. In [23] the authors studied a very general Ansatz for non-extremal deformations of $NS5$ branes wrapped on S^2. Their Ansatz can be adapted to our case. It reads –in Einstein frame,

$$ds^2 = -Y_1\,dt^2 + Y_2\,dx^ndx^n + Y_3\,d\rho^2 + Y_4\left(e_1^2 + e_2^2\right) + Y_5\left((w^1)^2 + (w^2)^2\right) + Y_6\left(w^3 + A\right)^2 \tag{F.1}$$

Inserting this ansatz in the supergravity action we get,

$$L = \sum_{i,j} G_{ij}(Y)Y_i'Y_j' - U(Y) = T - U \tag{F.2}$$

Using a parametrization to make G_{ij} diagonal and choosing the appropiate gauge to make contact with our ansatz we have,

$$
\begin{aligned}
T &= e^{2(g+q-4x+\Phi)}\frac{1}{8}\left(\frac{1}{2}\left(g'^2 + q'^2 - 2\Phi'^2\right) + 2(g'q' - k'x') + (g' + q' + \Phi')(k' - 4x' + 2\Phi')\right), \\
U &= \frac{1}{256}e^{-2(g+q-\Phi)}\left(-16e^{2(g+q+k)}(e^{2g} + 4e^{2q}) + (e^{4g} + 16e^{4q})(1 + e^{4k})\right).
\end{aligned}
\tag{F.3}
$$

and,

$$
\begin{aligned}
Y_1 &= e^{\Phi/2}e^{-8x}, &\quad Y_2 &= e^{\Phi/2}, &\quad Y_3 &= e^{\Phi/2}e^{8x}e^{2k}, \\
Y_4 &= e^{\Phi/2}e^{2g}, &\quad Y_5 &= e^{\Phi/2}e^{2q}. &\quad Y_6 &= \Phi
\end{aligned}
\tag{F.4}
$$

¿From (F.2) we get the second order equations to solve,

$$2e^{-4g+8x} + \frac{1}{8}e^{-4q+8x} - 2\Phi'(g' + q' - 4x' + \Phi') - \Phi'' = 0$$

$$-2x'(g' + q' - 4x' + \Phi') - x'' = 0$$

$$-e^{4k+8x}(2e^{-4g} + \frac{1}{8}e^{-4q}) + e^{8x}(2e^{-4g} + \frac{1}{8}e^{-4q}) + 2k'(g' + q' - 4x' + \Phi') + k'' = 0 \tag{F.5}$$

$$2e^{8x}(e^{4k-4g} - 2e^{2(k-g)} + 1) + g'(g' + q' - 4x' + \Phi') + g'' = 0$$

$$e^{8x}(-\frac{1}{8}e^{4k-4q} + e^{2k-2q} - \frac{1}{8}e^{-4q}) - 2q'(g' + q' - 4x' + \Phi') - q'' = 0$$

and a first order constraint which is a consequence of reparametrization invariance,

$$\frac{e^{8x}}{2}\left(\frac{1}{16}(e^{-4q} + 16e^{-4g})(1 + e^{4k}) - e^{2k}(e^{-2q} + 4e^{-2g})\right) + \frac{1}{2}(g'^2 + q'^2 - 2\Phi'^2) +$$

$$2(g'q' - k'x') + (g' + q' + \Phi')(k' - 4x' + 2\Phi') = 0 \tag{F.6}$$

The second equation in (F.5) can be integrated to yield a first order equation and (F.5) becomes

$$2e^{-4g+8x} + \frac{1}{8}e^{-4q+8x} - 2\Phi'(g' + q' - 4x' + \Phi') - \Phi'' = 0$$

$$x' - \alpha e^{-2(g+q-4x+\Phi)} = 0$$

$$-e^{4k+8x}(2e^{-4g} + \frac{1}{8}e^{-4q}) + e^{8x}(2e^{-4g} + \frac{1}{8}e^{-4q}) + 2k'(g' + q' - 4x' + \Phi') + k'' = 0 \tag{F.7}$$

$$e^{8x}(e^{4k-4g} - 2e^{2(k-g)} + 1) + g'(g' + q' - 4x' + \Phi') + g'' = 0$$

$$e^{8x}(-\frac{1}{8}e^{4k-4q} + e^{2k-2q} - \frac{1}{8}e^{-4q}) - 2q'(g' + q' - 4x' + \Phi') - q'' = 0$$

where α is a non-extremality parameter.

One can verify that the transformation

$$r \to e^{2d}r \qquad \Phi \to \Phi + d \qquad x \to x + \frac{d}{2} \qquad g \to g \qquad q \to q \qquad k \to k \qquad \text{(F.8)}$$

where d is a constant, leaves (F.5) and (F.6) invariant and thus, is a symmetry of the equations of motion and constraint. Another, obvious, symmetry of (F.5) and (F.6) is

$$\Phi \to \Phi + C \qquad \alpha \to \alpha e^{2C} \qquad \text{(F.9)}$$

with all the other functions unchanged and C a constant. Finally, there is also a translational symmetry

$$r \to r + r_0 \qquad \text{(F.10)}$$

that leaves all equations unchanged. Note that thermodynamic quantities should be invariant under all these symmetries.

References

[1] J. M. Maldacena, Adv. Theor. Math. Phys. **2**, 231 (1998) [Int. J. Theor. Phys. **38**, 1113 (1999)] [arXiv:hep-th/9711200].

[2] S. S. Gubser, I. R. Klebanov and A. M. Polyakov, Phys. Lett. B **428**, 105 (1998) [arXiv:hep-th/9802109]. E. Witten, Adv. Theor. Math. Phys. **2**, 253 (1998) [arXiv:hep-th/9802150]. N. Itzhaki, J. M. Maldacena, J. Sonnenschein and S. Yankielowicz, Phys. Rev. D **58**, 046004 (1998) [arXiv:hep-th/9802042].

[3] O. Lunin and J. M. Maldacena, JHEP **0505**, 033 (2005) [arXiv:hep-th/0502086]. J. Maldacena, D. Martelli and Y. Tachikawa, JHEP **0810**, 072 (2008) [arXiv:0807.1100 [hep-th]]. C. P. Herzog, M. Rangamani and S. F. Ross, JHEP **0811**, 080 (2008) [arXiv:0807.1099 [hep-th]]. C. P. Herzog, M. Rangamani and S. F. Ross, JHEP **0811**, 080 (2008) [arXiv:0807.1099 [hep-th]].

[4] V. Belinski and E. Verdaguer, *Cambridge, UK: Univ. Pr. (2001) 258 p*A. A. Pomeransky, Phys. Rev. D **73**, 044004 (2006) [arXiv:hep-th/0507250]. T. Mishima and H. Iguchi, Phys. Rev. D **73**, 044030 (2006) [arXiv:hep-th/0504018].

[5] H. Elvang and P. Figueras, JHEP **0705**, 050 (2007) [arXiv:hep-th/0701035].

[6] J. Maldacena and D. Martelli, JHEP **1001**, 104 (2010) [arXiv:0906.0591 [hep-th]].

[7] A. Butti, M. Grana, R. Minasian, M. Petrini and A. Zaffaroni, JHEP **0503**, 069 (2005) [arXiv:hep-th/0412187].

[8] I. R. Klebanov and M. J. Strassler, JHEP **0008**, 052 (2000) [arXiv:hep-th/0007191].

[9] R. Casero, C. Nunez and A. Paredes, Phys. Rev. D **73**, 086005 (2006) [arXiv:hep-th/0602027]. R. Casero, C. Nunez and A. Paredes, Phys. Rev. D **77**, 046003 (2008) [arXiv:0709.3421 [hep-th]].

[10] C. Hoyos-Badajoz, C. Nunez and I. Papadimitriou, Phys. Rev. D **78**, 086005 (2008) [arXiv:0807.3039 [hep-th]].

[11] A. H. Chamseddine and M. S. Volkov, Phys. Rev. Lett. **79**, 3343 (1997) [arXiv:hep-th/9707176]. J. M. Maldacena and C. Nunez, Phys. Rev. Lett. **86**, 588 (2001) [arXiv:hep-th/0008001].

[12] R. Minasian, M. Petrini and A. Zaffaroni, JHEP **1004**, 080 (2010) [arXiv:0907.5147 [hep-th]].

[13] J. Gaillard, D. Martelli, C. Nunez and I. Papadimitriou, Nucl. Phys. B **843**, 1 (2011) [arXiv:1004.4638 [hep-th]].

[14] N. Halmagyi, arXiv:1003.2121 [hep-th].

[15] A. Dymarsky, I. R. Klebanov and N. Seiberg, JHEP **0601**, 155 (2006) [arXiv:hep-th/0511254].

[16] R. P. Andrews and N. Dorey, Nucl. Phys. B **751**, 304 (2006) [arXiv:hep-th/0601098]. R. P. Andrews and N. Dorey, Phys. Lett. B **631**, 74 (2005) [arXiv:hep-th/0505107].

[17] E. Witten, Adv. Theor. Math. Phys. **2**, 505 (1998) [arXiv:hep-th/9803131].

[18] T. Sakai and S. Sugimoto, Prog. Theor. Phys. **113**, 843 (2005) [arXiv:hep-th/0412141].

[19] O. Aharony, J. Sonnenschein and S. Yankielowicz, Annals Phys. **322**, 1420 (2007) [arXiv:hep-th/0604161].

[20] J. Gaillard and D. Martelli, arXiv:1008.0640 [hep-th].

[21] C. Nunez, A. Paredes and A. V. Ramallo, Adv. High Energy Phys. **2010**, 196714 (2010) [arXiv:1002.1088 [hep-th]].

[22] C. Nunez, I. Papadimitriou and M. Piai, Int. J. Mod. Phys. A **25**, 2837 (2010) [arXiv:0812.3655 [hep-th]].

[23] S. S. Gubser, A. A. Tseytlin, and M. S. Volkov, *JHEP* **0109:017,2001** (JHEP 0109:017,2001).

[24] E. Caceres, R. Flauger, M. Ihl, and T. Wrase, *JHEP* **0803:020,2008** (Nov., 2007).

[25] E. Caceres, R. Flauger, and T. Wrase, arXiv:0908.4483.

[26] G. T. Horowitz, A. Strominger, Nucl. Phys. **B360**, 197-209 (1991).

[27] M. J. Duff, J. X. Lu, Nucl. Phys. **B416**, 301-334 (1994). [hep-th/9306052].

[28] M. J. Duff, R. R. Khuri, J. X. Lu, Phys. Rept. **259**, 213-326 (1995). [hep-th/9412184].

[29] M. Atiyah, J. M. Maldacena and C. Vafa, J. Math. Phys. **42** (2001) 3209 [arXiv:hep-th/0011256]. C. Vafa, J. Math. Phys. **42**, 2798 (2001) [arXiv:hep-th/0008142]. J. D. Edelstein and C. Nunez, JHEP **0104**, 028 (2001) [arXiv:hep-th/0103167].

[30] A. Brandhuber, Nucl. Phys. B **629**, 393 (2002) [arXiv:hep-th/0112113].

[31] J. M. Maldacena and H. S. Nastase, JHEP **0109**, 024 (2001) [arXiv:hep-th/0105049].

[32] C. G. . Callan, J. A. Harvey and A. Strominger, Nucl. Phys. B **359** (1991) 611.

[33] A. Buchel, Nucl. Phys. B **600** (2001) 219 [arXiv:hep-th/0011146]. A. Buchel, C. P. Herzog, I. R. Klebanov, L. A. Pando Zayas and A. A. Tseytlin, JHEP **0104** (2001) 033 [arXiv:hep-th/0102105]. S. S. Gubser, C. P. Herzog, I. R. Klebanov and A. A. Tseytlin, JHEP **0105** (2001) 028 [arXiv:hep-th/0102172]. L. A. Pando Zayas and C. A. Terrero-Escalante, JHEP **0609** (2006) 051 [arXiv:hep-th/0605170]. O. Aharony, A. Buchel and P. Kerner, Phys. Rev. D **76** (2007) 086005 [arXiv:0706.1768 [hep-th]]. M. Mahato, L. A. Pando Zayas and C. A. Terrero-Escalante, JHEP **0709** (2007) 083 [arXiv:0707.2737 [hep-th]]. A. Buchel, arXiv:1012.2404 [hep-th].

[34] G. T. Horowitz and D. L. Welch, Phys. Rev. D **49**, 590 (1994) [arXiv:hep-th/9308077].

[35] M. Schvellinger, JHEP **0409**, 057 (2004) [arXiv:hep-th/0407152]. P. McGuirk, G. Shiu and Y. Sumitomo, Nucl. Phys. B **842**, 383 (2010) [arXiv:0910.4581 [hep-th]].

[36] I. Bena, M. Grana and N. Halmagyi, JHEP **1009**, 087 (2010) [arXiv:0912.3519 [hep-th]]. I. Bena, G. Giecold and N. Halmagyi, arXiv:1011.2195 [hep-th].

[37] J. Gaillard and J. Schmude, JHEP **1002**, 032 (2010) [arXiv:0908.0305 [hep-th]].

[38] F. Bigazzi, A. L. Cotrone, J. Mas, A. Paredes, A. V. Ramallo and J. Tarrio, JHEP **0911**, 117 (2009) [arXiv:0909.2865 [hep-th]]. F. Bigazzi, A. L. Cotrone and J. Tarrio, JHEP **1002**, 083 (2010) [arXiv:0912.3256 [hep-th]]. F. Bigazzi and A. L. Cotrone, JHEP **1008**, 128 (2010) [arXiv:1006.4634 [hep-ph]]. M. Mia, K. Dasgupta, C. Gale and S. Jeon, Nucl. Phys. B **839** (2010) 187 [arXiv:0902.1540 [hep-th]]. F. Bigazzi, A. L. Cotrone, J. Mas, D. Mayerson and J. Tarrio, arXiv:1101.3560 [hep-th].

[39] F. Chen, K. Dasgupta, P. Franche, S. Katz and R. Tatar, arXiv:1007.5316 [hep-th].